Universally meager spaces and a conjecture of Galvin

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Program

Determine the optimal class of topological spaces X with the property that for all integers $k, \ell \geq 2$,

$$X \to (\mathrm{top}\mathbb{Q})^{k}_{\ell,\ k!(k-1)!}.$$

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The following are equivalent for a metrizable space X:

- *X is not σ-discrete.*
- *$X \to (\mathrm{top}\mathbb{Q})^{k}_{\ell,\ k!(k-1)!}$ for all integers $k, \ell \geq 2$.*

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Program

Determine the optimal class if topological space X for which the following two conditions are equivalent:

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The following are equivalent for a topological space X:

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Corollary (T., 2007)

If there is a compact cardinal other than ω, then every universally meager metrizable space X is σ-discrete.

Back to the Ramsey degree conjecture

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Theorem (Raghavan-T., 2017/2018)

If there is a compact cardinal other than ω, then the following are equivalent for every space X with point countable base:

- *X is not left separated.*
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Corollary (Raghavan-T., 2017/2018)

If there is a compact cardinal other than ω*, then the following are equivalent for every metrizable space* X*:*

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