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A fragment of Asperó-Mota's Finitely Proper Forcing Axiom and an entangled set of reals

We introduce a fragment PFA^{s-fin}(ω_1) of Asperó-Mota's finitely proper forcing axiom PFA^{fin}(ω_1). PFA^{s-fin}(ω_1) implies some consequences of PFA^{fin}(ω_1), for example MA_{\aleph_1}, the failure of \mho , no weak club guessing ladder systems, and the assertion that every two Aronszajn trees are club-isomorphic. For each integer $k \ge 2$, it is consistent that PFA^{s-fin}(ω_1) holds, there exists a k-entangled set of reals, and $2^{\aleph_0} = \aleph_2$. This extends Abraham-Shelah's theorem that Martin's axiom does not imply that every two \aleph_1 -dense sets of reals are isomorphic.