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Open colorings, perfect sets and games on generalized Baire spaces

The generalized Baire space for an uncountable cardinal $\kappa = \kappa^{<\kappa}$ is the space ${}^{\kappa}\kappa$ of functions $\kappa \to \kappa$ equipped with the $<\kappa$-support topology. The study of the topology and descriptive set theory of these spaces is an active area of research today, with close connections to many other areas of set theory and to model theory.

The notions of perfectness, scatteredness and the Cantor-Bendixson hierarchy were generalized for the κ-Baire space ${}^{\kappa}\kappa$ by J. Väänänen [1] based on games of length $\leq \kappa$. Some different definitions of κ-perfectness for ${}^{\kappa}\kappa$ are also widely used. In the classical setting, these definitions correspond to equivalent notions, but this is no longer the case in the uncountable setting. In the first part of this talk, we detail connections between these concepts and their underlying games. For example, we show that Väänänen's Cantor-Bendixson theorem [1] is equivalent to the κ-perfect set property, and is therefore equiconsistent with the existence of an inaccessible cardinal above κ.

In the second part of this talk we introduce the uncountable analogue $\mathrm{OCA}_{\kappa}(X)$ of the Open Coloring Axiom for subsets X of the κ-Baire space, and also its κ-perfect set version $\mathrm{OCA}^*_{\kappa}(X)$. We show that $\mathrm{OCA}^*_{\kappa}(X)$ for all κ-analytic subsets $X \subseteq {}^{\kappa}\kappa$ is consistent relative to (and therefore equiconsistent with) the existence of an inaccessible cardinal above κ.

[1] Väänänen, J., A Cantor-Bendixson theorem for the space $\omega_1^{\omega_1}$. Fundamenta Mathematicae, Vol. 137 (1991), 187–199.