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Nonamalgamation in the generic multiverse

Joint work with Joel David Hamkins, Lukas Daniel Klausner, Jonathan Verner, and Kameryn Williams

Fix a countable transitive model M of ZFC and consider its generic multiverse, the family of all forcing extensions of M. If we order the generic multiverse by inclusion, the resulting structure has interesting universality properties. Mostowski showed that any finite poset embeds into the generic multiverse in a way that also preserves *nonamalgamability*, i.e. the nonexistence of upper bounds. His embedding actually only used extensions adding Cohen reals. I will present some results from a joint project in which we extended Mostowski's result in order to embed several infinite posets into the generic multiverse. In addition to nonamalgamability, our method also ensures that greatest lower bounds are preserved. In particular, any finite meet-semilattice embeds as such into the generic multiverse. Further variations on the proof also allow us to realize these embeddings with a wide variety of forcing notions beyond just Cohen forcing.