

The spectrum of independence

The set of possible sizes of maximal independent families is referred to as the spectrum of independence and denoted $\text{Sp}(\mathfrak{i})$. We will show that:

- Whenever $\{\kappa_i\}_{i=1}^n$ are regular uncountable cardinals, it is consistent that $\{\kappa_i\}_{i=1}^n \subseteq \text{Sp}(\mathfrak{i})$.
- Whenever κ has uncountable cofinality, it is consistent that $\text{Sp}(\mathfrak{i}) = \{\aleph_1, \kappa = \mathfrak{c}\}$.
- Assuming that $\kappa_1 < \dots < \kappa_n$ are measurable cardinals, it is consistent that

$$\{\kappa_i\}_{i=1}^n \subseteq \text{Sp}(\mathfrak{i}) \text{ and } \left(\bigcup_{i=1}^{n-1} (\kappa_i, \kappa_{i+1}) \right) \cap \text{Sp}(\mathfrak{i}) = \emptyset.$$

In addition, to any independent family, we will associate two ideals on ω and define a class of maximal independent families for which Sacks indestructibility can be naturally characterized in terms of these ideals.