MatTriad 2015, Coimbra

H-matrix theory and applications

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joint work with Ljiljana Cvetković

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- H-matrices and SDD-property
- Benefits from H-subclasses

- Breaking the SDD
- Additive and multiplicative conditions
- Partitioning the index set
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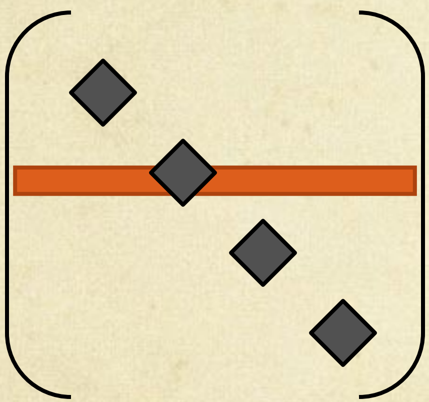
- Nonstrict conditions

H-matrices and SDD-property

A complex matrix $A=[a_{ij}]_{n \times n}$ is an SDD-matrix if for each *i* from N it holds that

$$|a_{ii}| > r_i(A) = \sum_{j \in N, j \neq i} |a_{ij}|$$

Deleted row sums

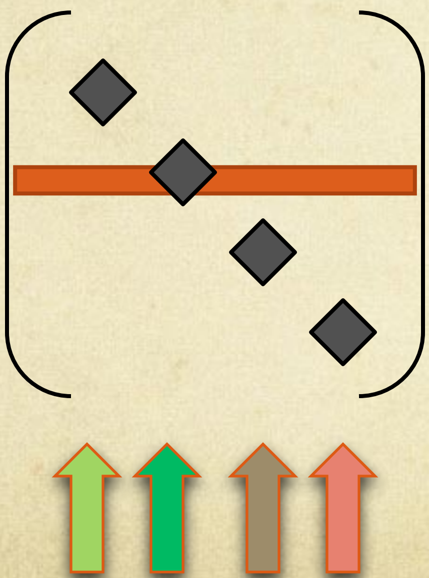


Lévy-Desplanques:
nonsingular

H-matrices and SDD-property

A complex matrix $A=[a_{ij}]_{nxn}$ is an SDD-matrix if for each *i* from N it holds that

$$|a_{ii}| > r_i(A) = \sum_{j \in N, j \neq i} |a_{ij}|$$

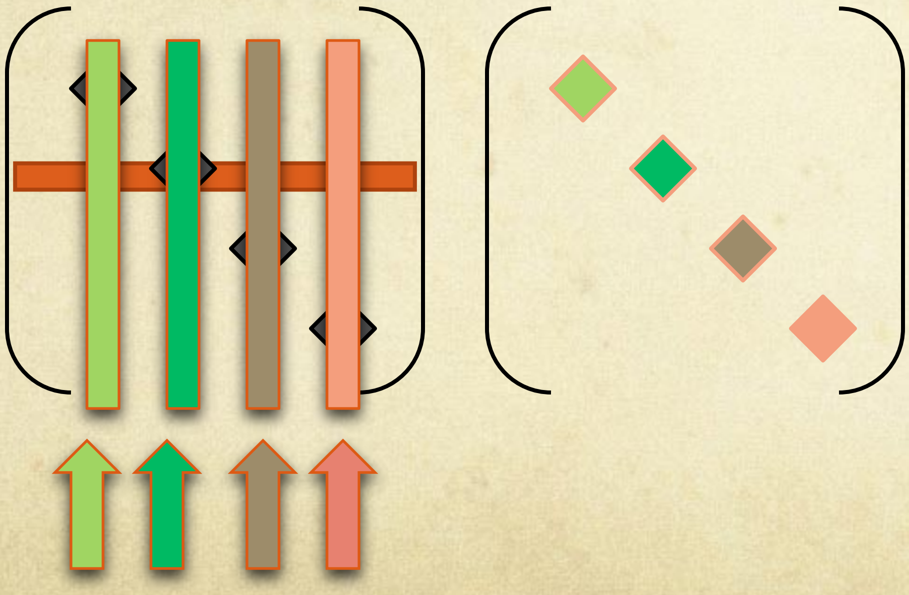


A complex matrix $A=[a_{ij}]_{nxn}$ is an H-matrix if and only if there exists a diagonal nonsingular matrix W such that AW is an SDD matrix.

H-matrices and SDD-property

A complex matrix $A=[a_{ij}]_{n \times n}$ is an SDD-matrix if for each *i* from N it holds that

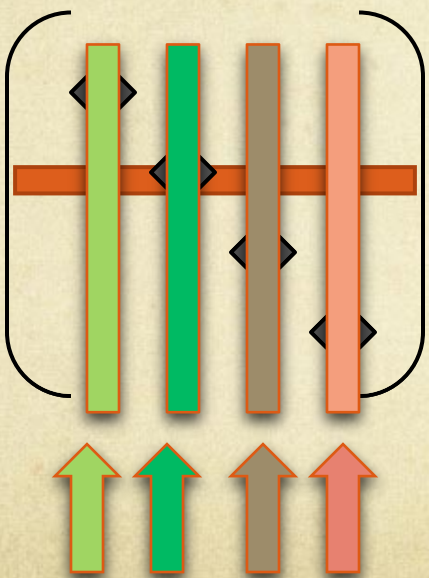
$$|a_{ii}| > r_i(A) = \sum_{j \in N, j \neq i} |a_{ij}|$$

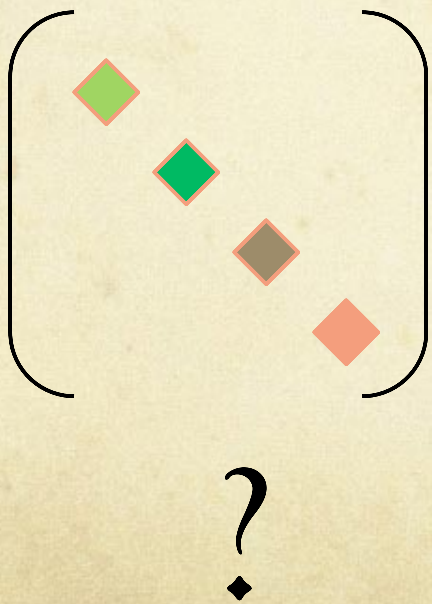


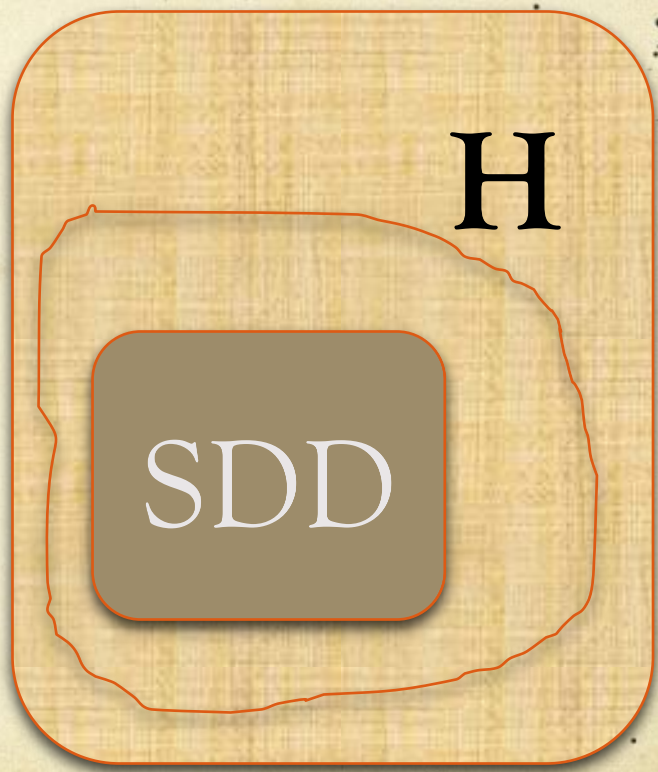
H-matrices and SDD-property

A complex matrix $A=[a_{ij}]_{n \times n}$ is an SDD-matrix if for each *i* from N it holds that

$$|a_{ii}| > r_i(A) = \sum_{j \in N, j \neq i} |a_{ij}|$$



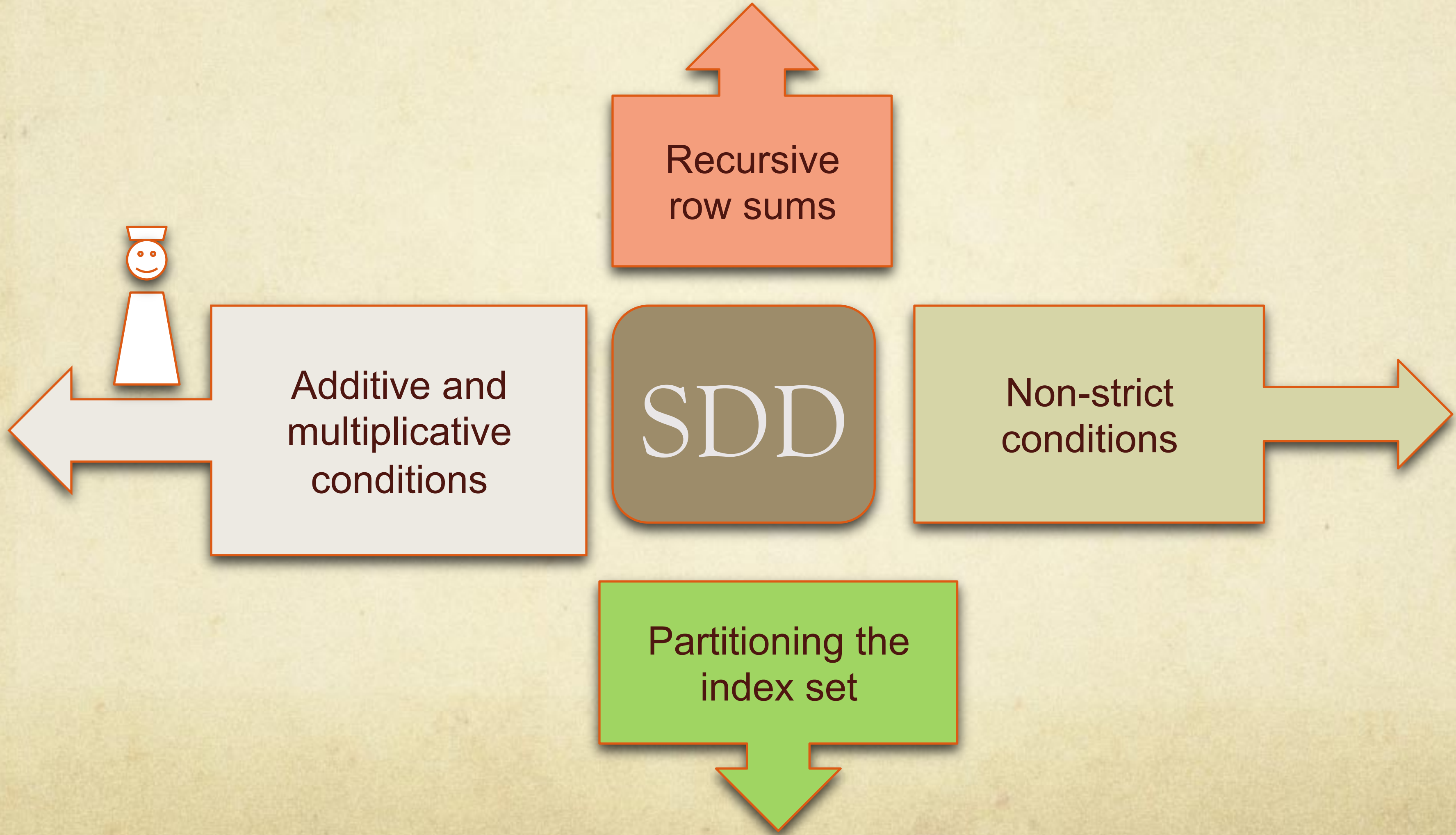




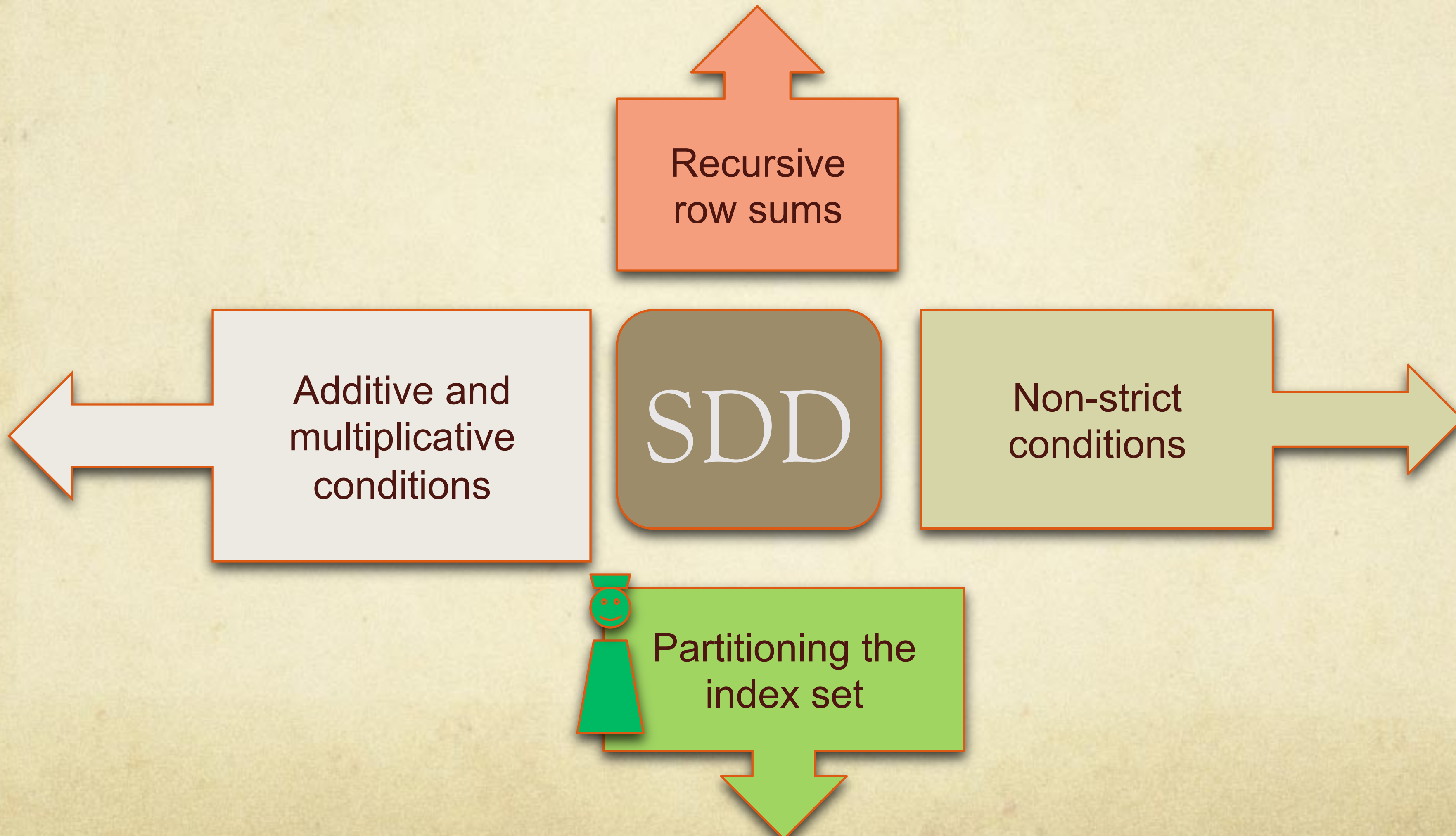
Subclasses of H-matrices & diagonal scaling characterizations Benefits:

1. Nonsingularity result covering a wider matrix class
2. A tighter eigenvalue inclusion area (not just for the observed class)
3. A new bound for the max-norm of the inverse for a wider matrix class
4. A tighter bound for the max-norm of the inverse for some SDD matrices
5. Schur complement related results (closure and eigenvalues)
6. Convergence area for relaxation iterative methods
7. Sub-direct sums
8. Bounds for determinants

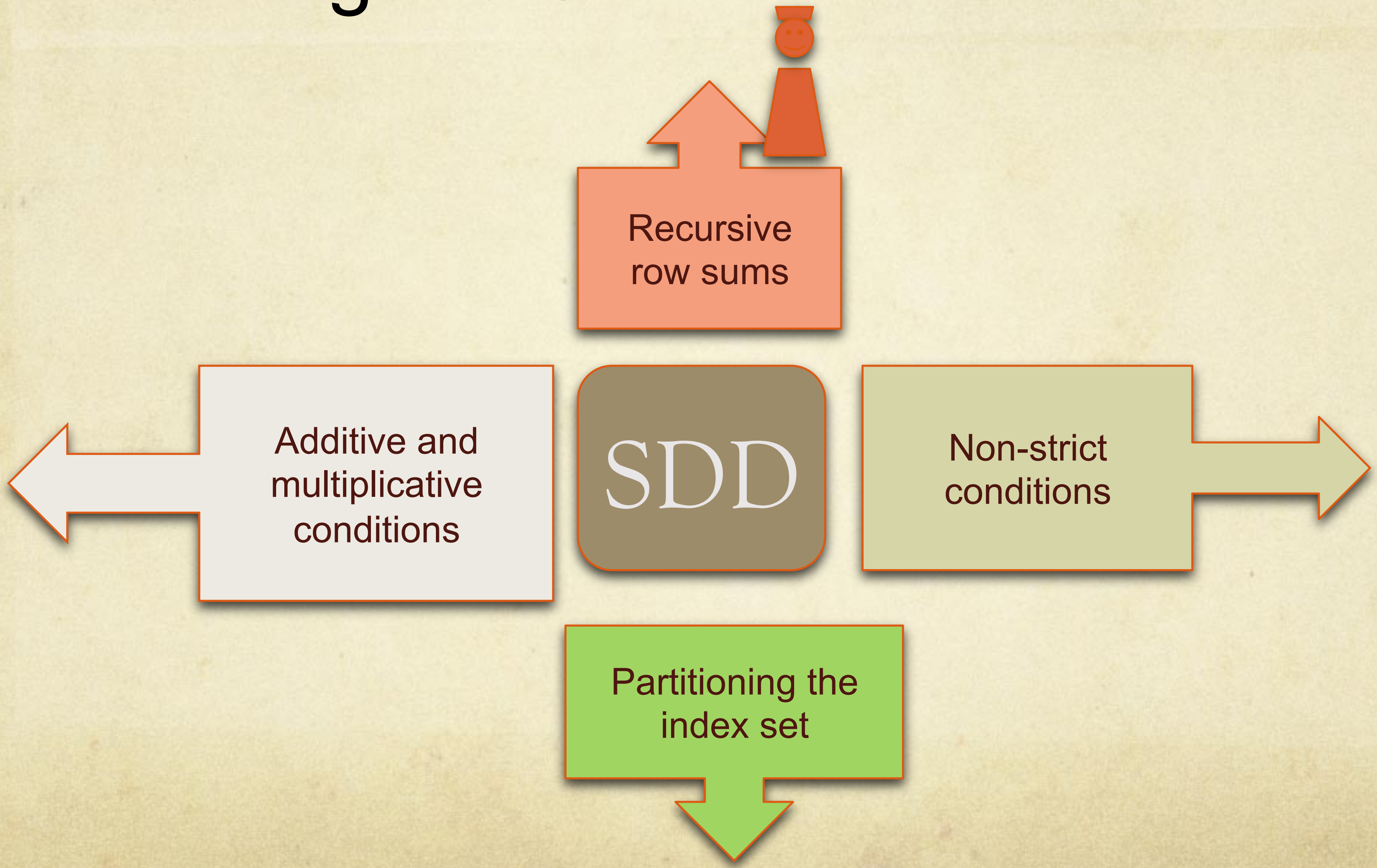
Breaking the SDD



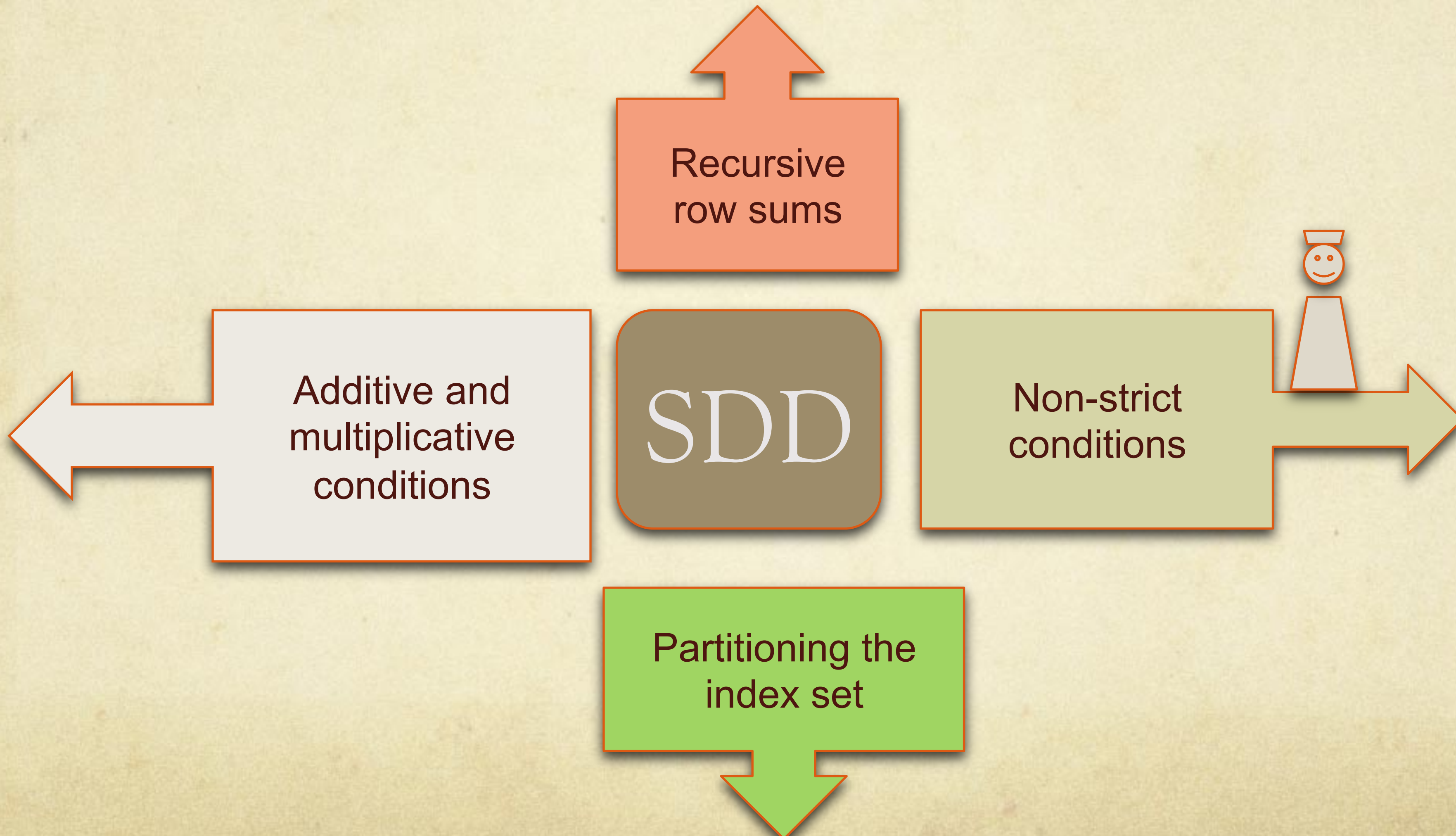
Breaking the SDD



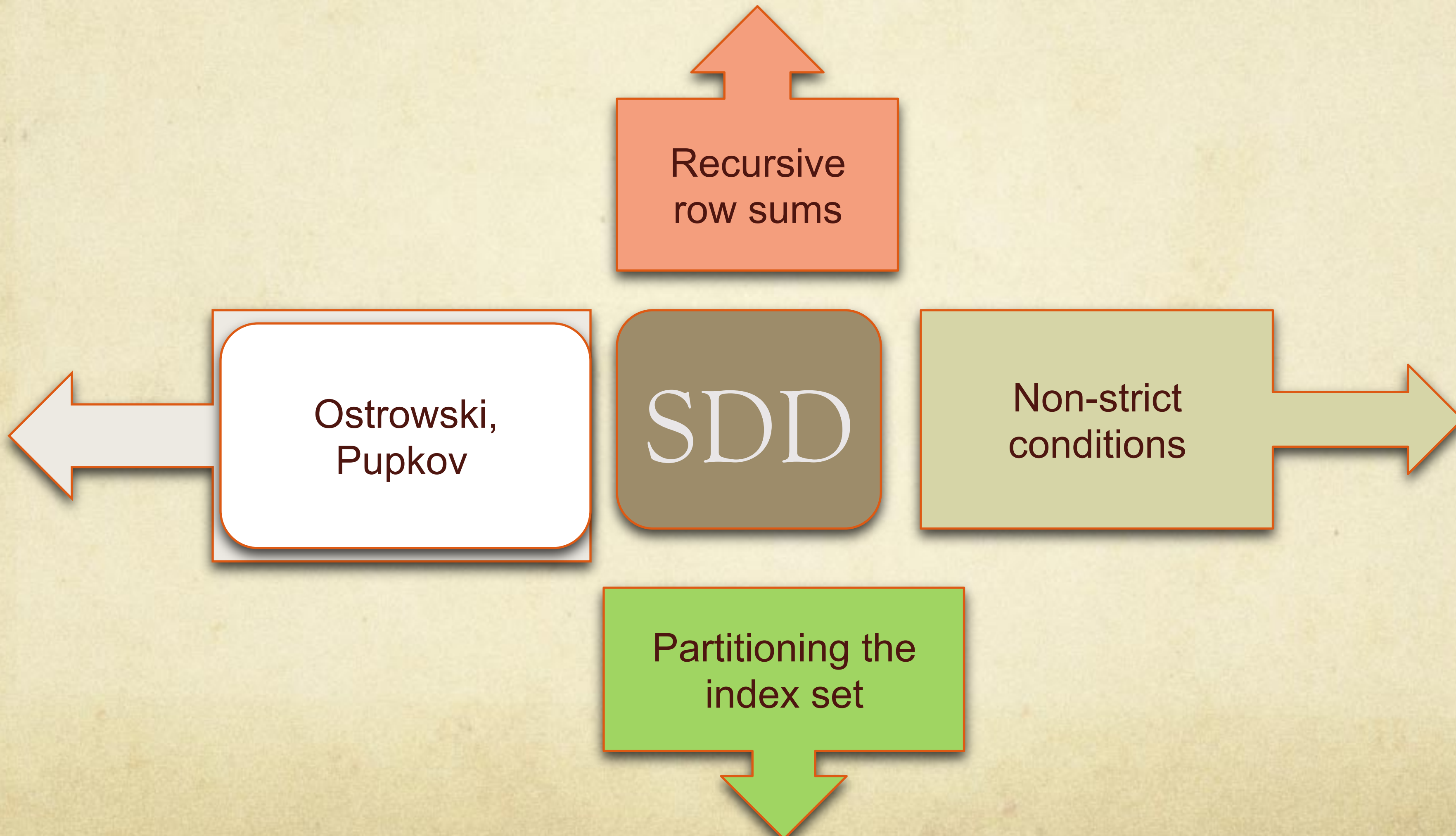
Breaking the SDD



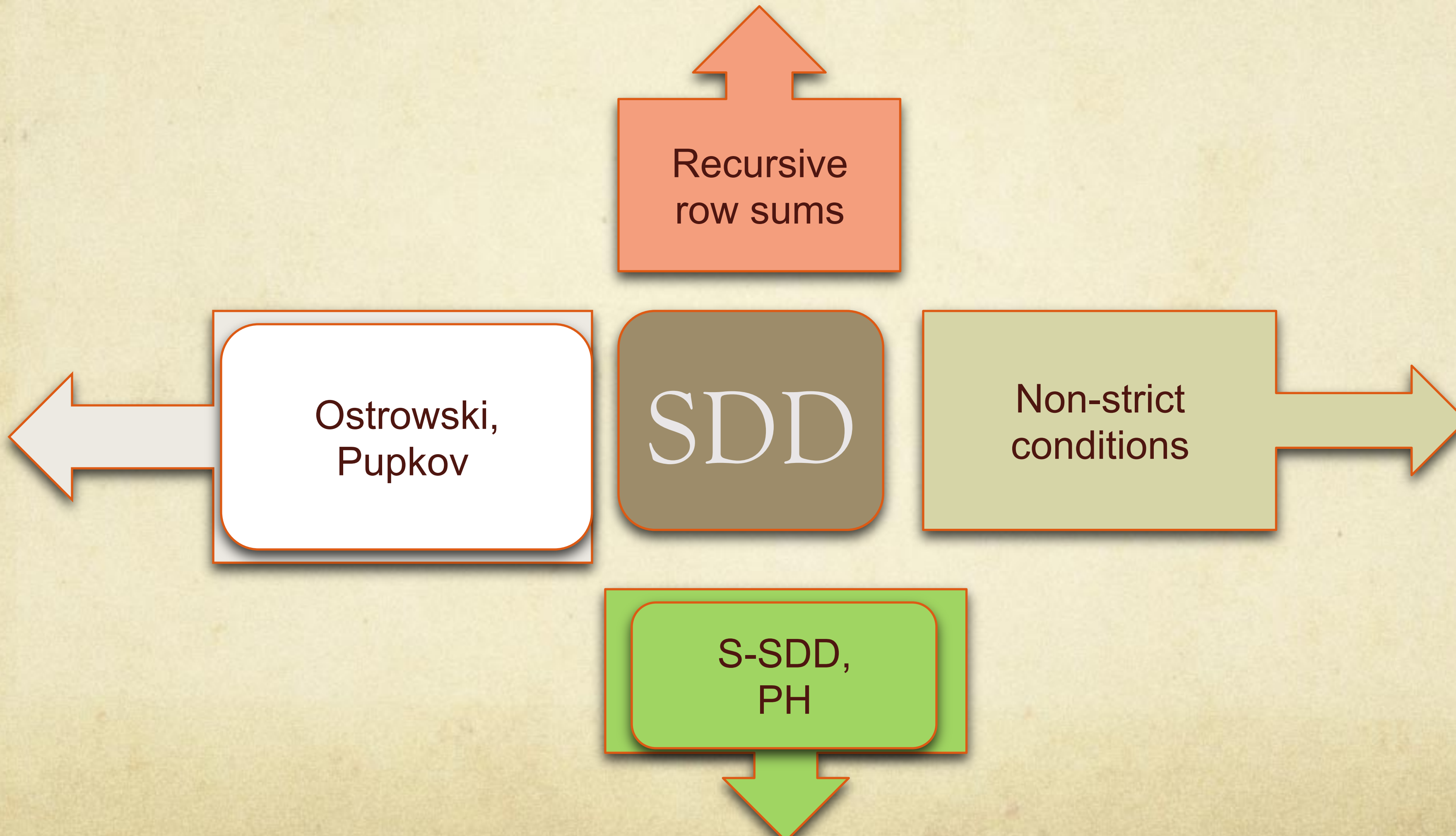
Breaking the SDD



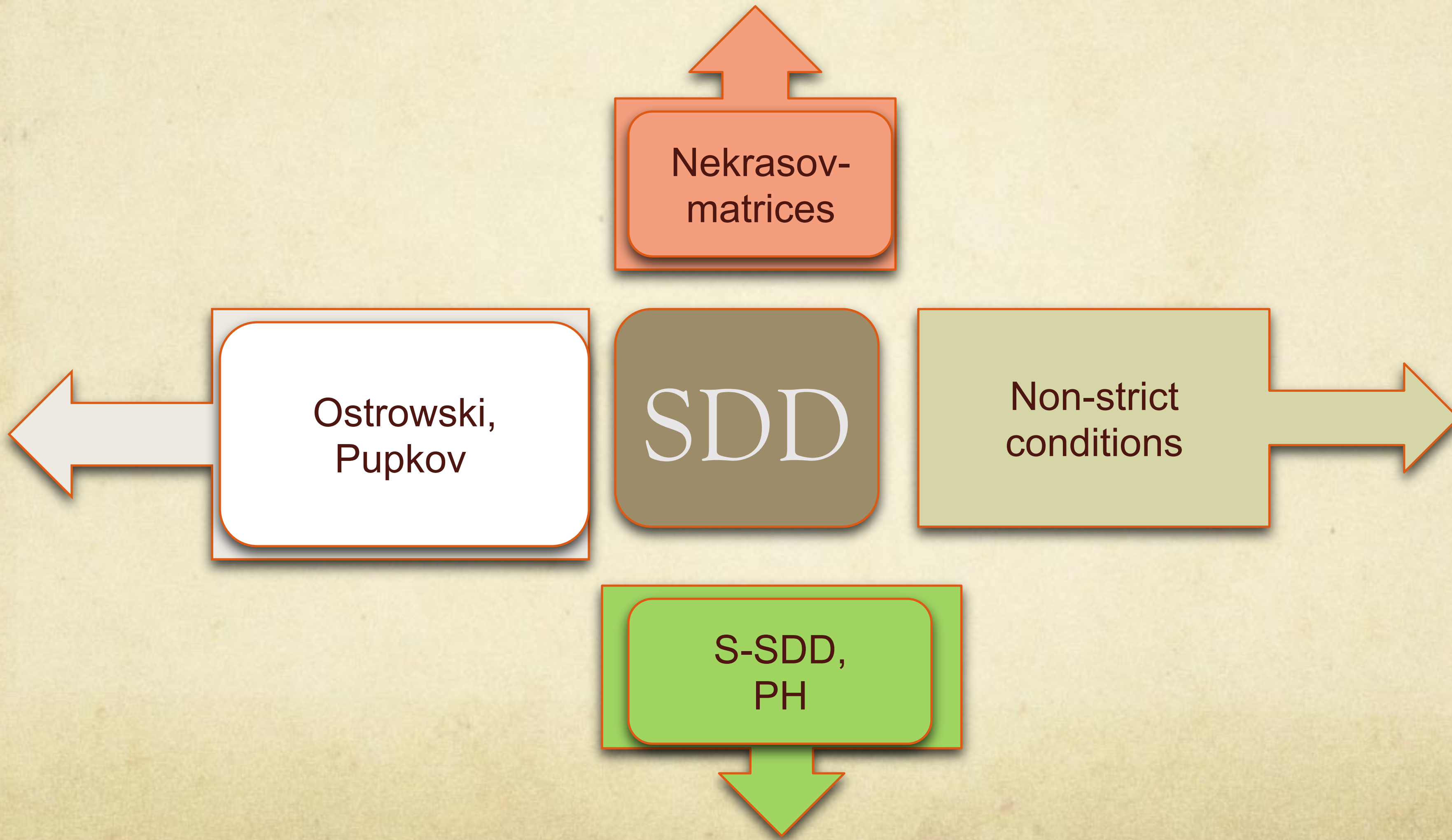
Ostrowski, A. M. (1937), Pupkov, V. A. (1983), Hoffman, A.J. (2000), Varga, R.S. : Geršgorin and his circles (2004)



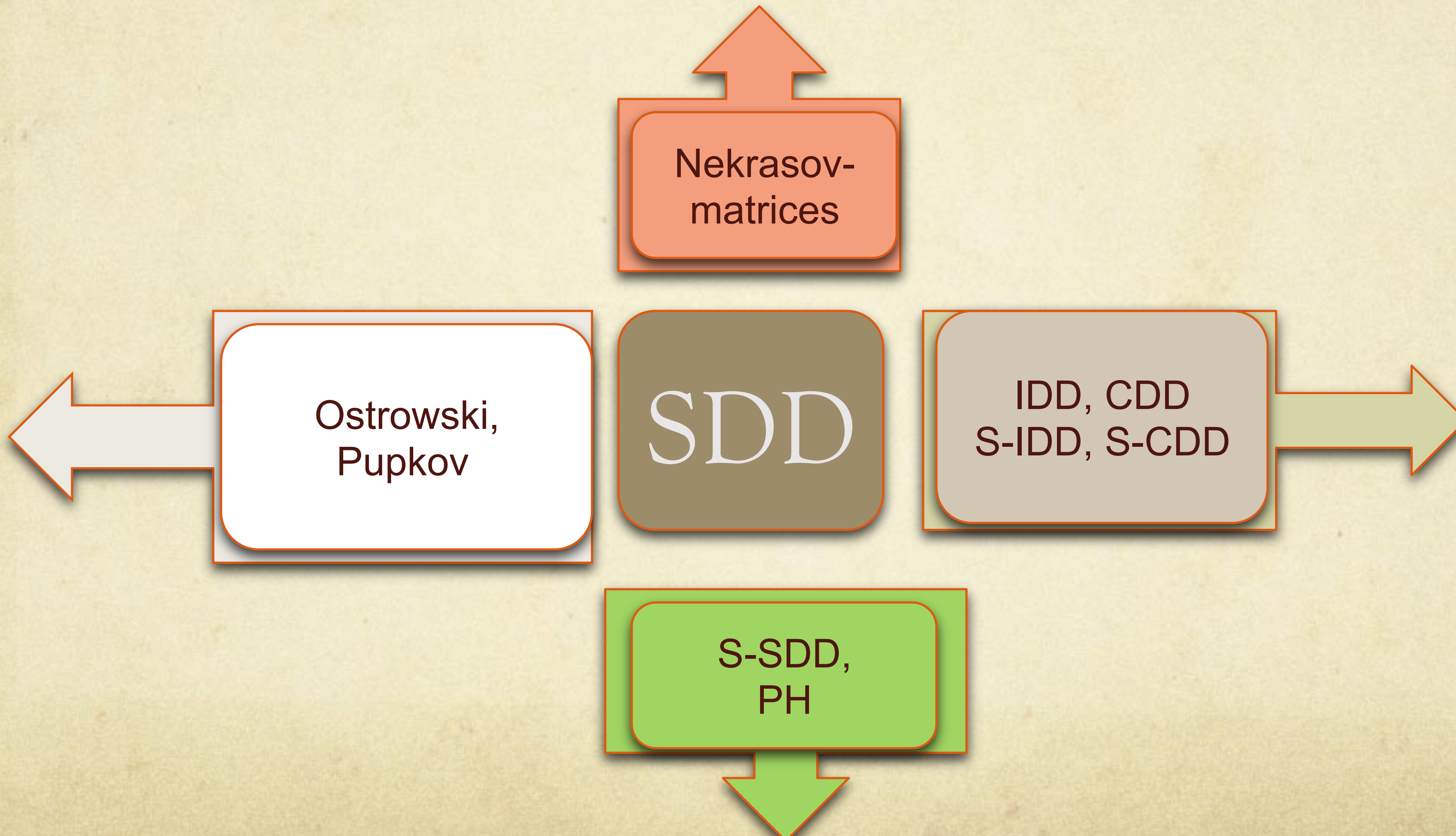
Gao, Y.M., Xiao, H.W. (1992), Varga, R.S. (2004),
Dashnic, L.S., Zusmanovich, M.S. (1970), Kolotilina, I. Yu.(2010),
Cvetković, Lj., Nedović, M. (2009), (2012), (2013).



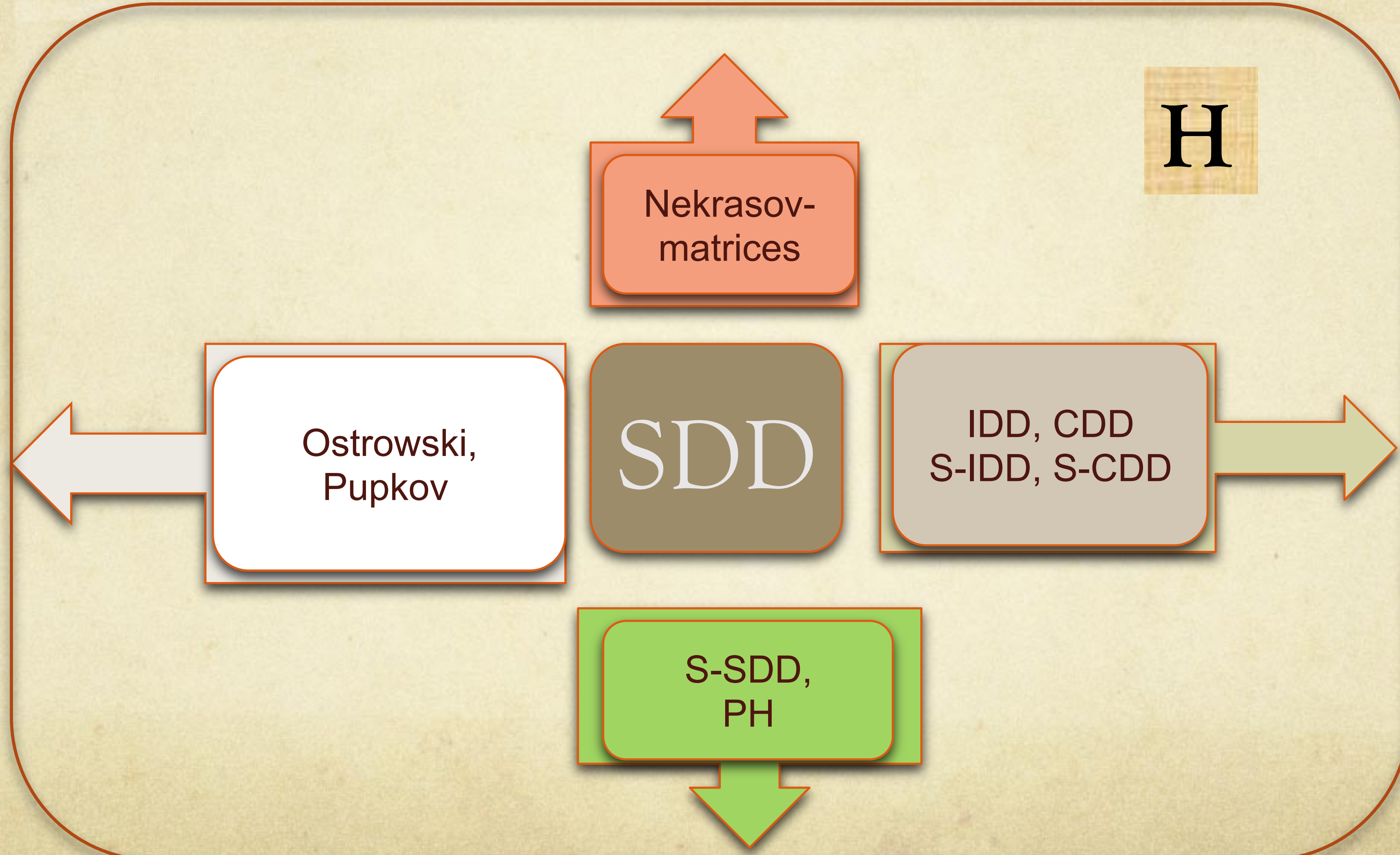
Mehmke, R., Nekrasov, P. (1892), Gudkov, V.V. (1965), Szulc, T. (1995), Li, W. (1998), Cvetković, Lj., Kostić, V., Nedović, M. (2014).



O. Taussky (1948), Beauwens (1976), Szulc,T. (1995), Li, W. (1998), Varga, R.S. (2004) Cvetković, Lj., Kostić, V. (2005)



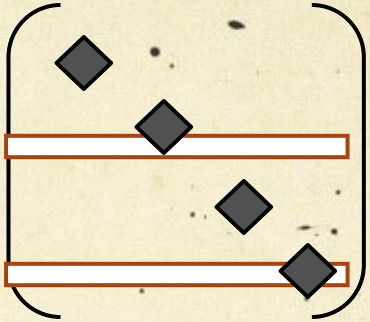
O. Taussky (1948), Beauwens (1976), Szulc,T. (1995), Li, W. (1998), Varga, R.S. (2004) Cvetković, Lj., Kostić, V. (2005)



I Additive and multiplicative conditions

Ostrowski-matrices
multiplicative condition:

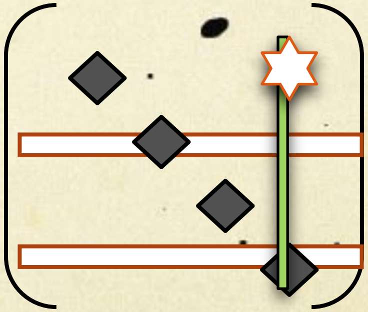
$$|a_{ii}||a_{jj}| > r_i(A) r_j(A)$$



Pupkov-matrices
additive condition:

$$|a_{ii}| > \min\{\max_{j \neq i}\{|a_{ji}|\}, r_i(A)\}$$

$$|a_{ii}| + |a_{jj}| > r_i(A) + r_j(A)$$



Ostrowski, A. M. (1937), Pupkov, V. A. (1983), Hoffman, A.J. (2000),
Varga, R.S. : Geršgorin and his circles (2004)

II Partitioning the index set

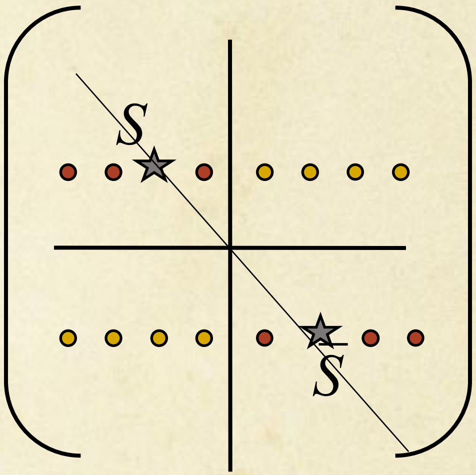
S-SDD-matrices

Given any complex matrix $A=[a_{ij}]_{n\times n}$ and given any nonempty proper subset S of N, A is an S-SDD matrix if

$$|a_{ii}| > r_i^S(A) = \sum_{j\in S,\, j\neq i} |a_{ij}|, \qquad i\in S$$

$$\left(|a_{ii}| - r_i^S(A)\right)\left(|a_{jj}| - r_j^{\bar{S}}(A)\right) > r_i^{\bar{S}}(A)\, r_j^S(A),$$

$$i\in S,\ j\in \bar{S}$$



Gao, Y.M., Xiao, H.W. LAA (1992)

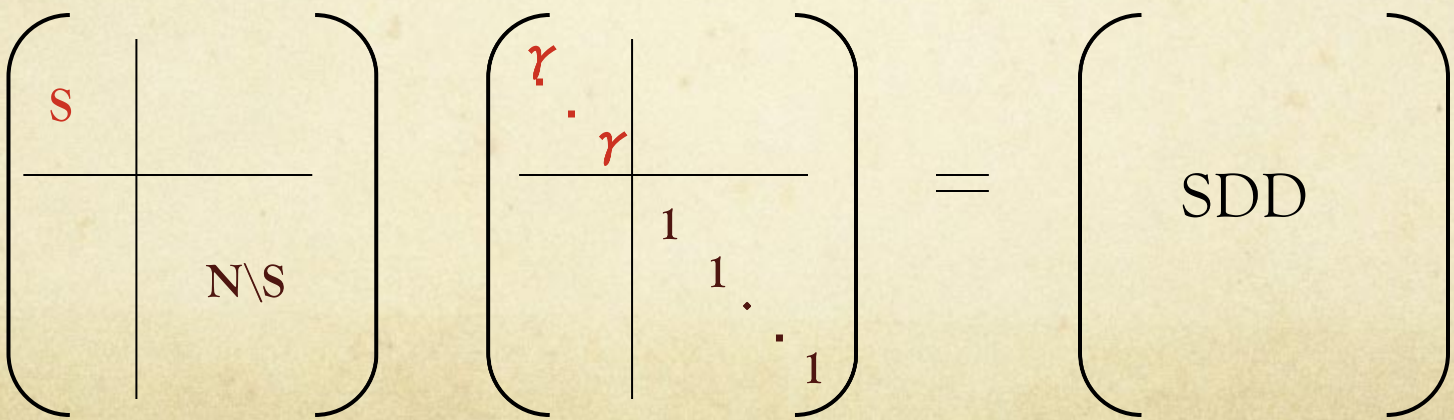
Cvetković, Lj., Kostić, V., Varga, R. ETNA (2004)

II Partitioning the index set

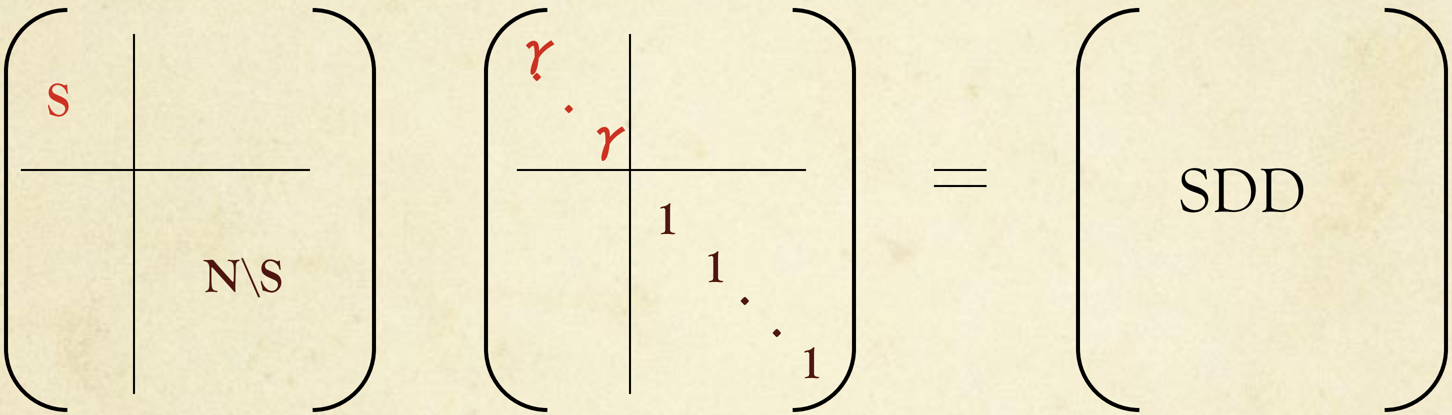
S-SDD-matrices

- A matrix $A=[a_{ij}]_{n\times n}$ is an S-SDD matrix iff there exists a matrix W in $\mathcal{W}^S$ such that AW is an SDD matrix.

$$\boldsymbol{W}^S = \left\{ W = diag(w_1, w_2, \dots, w_n) : w_i = \gamma > 0 \quad for \quad i \in S \quad and \quad w_i = 1 \quad for \quad i \in \bar{S} \right\}$$



Diagonal scaling characterization & Scaling matrices

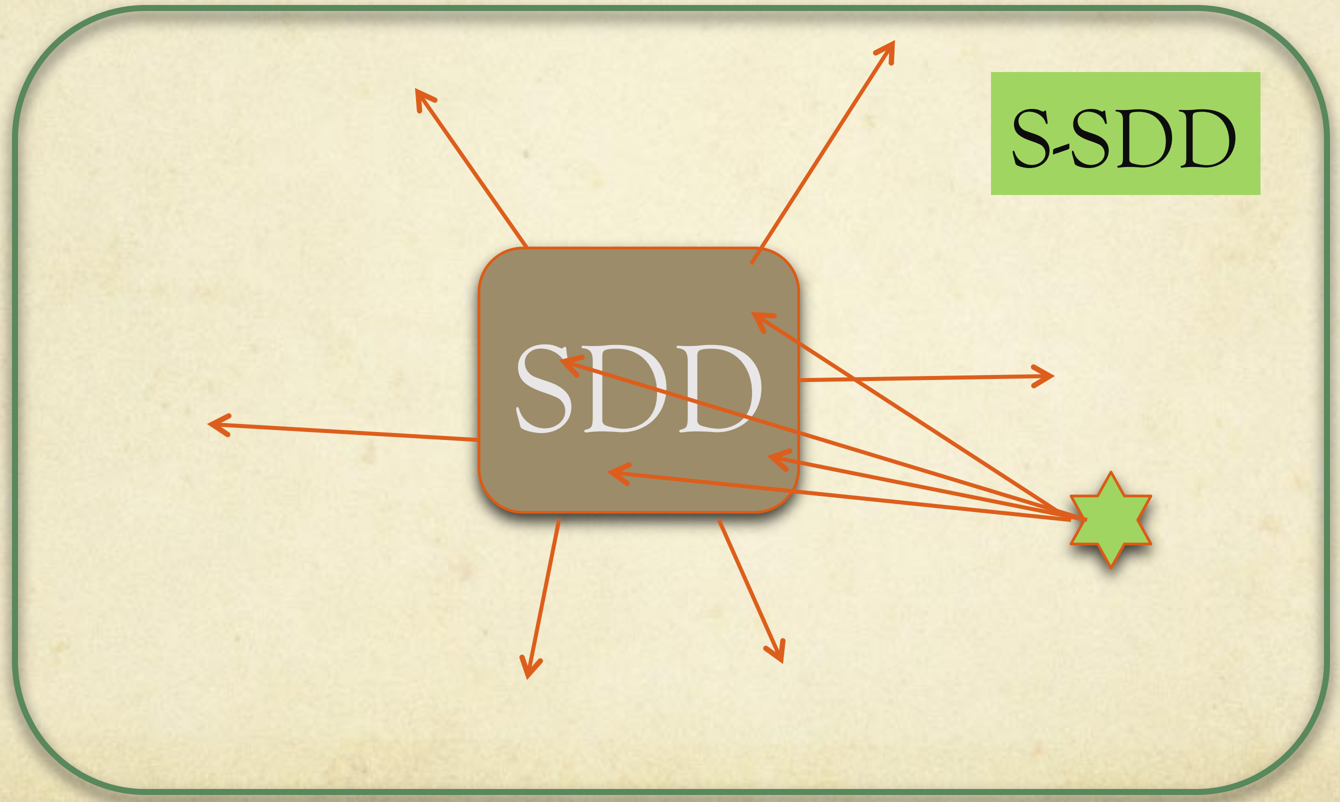


We choose parameter from the interval:

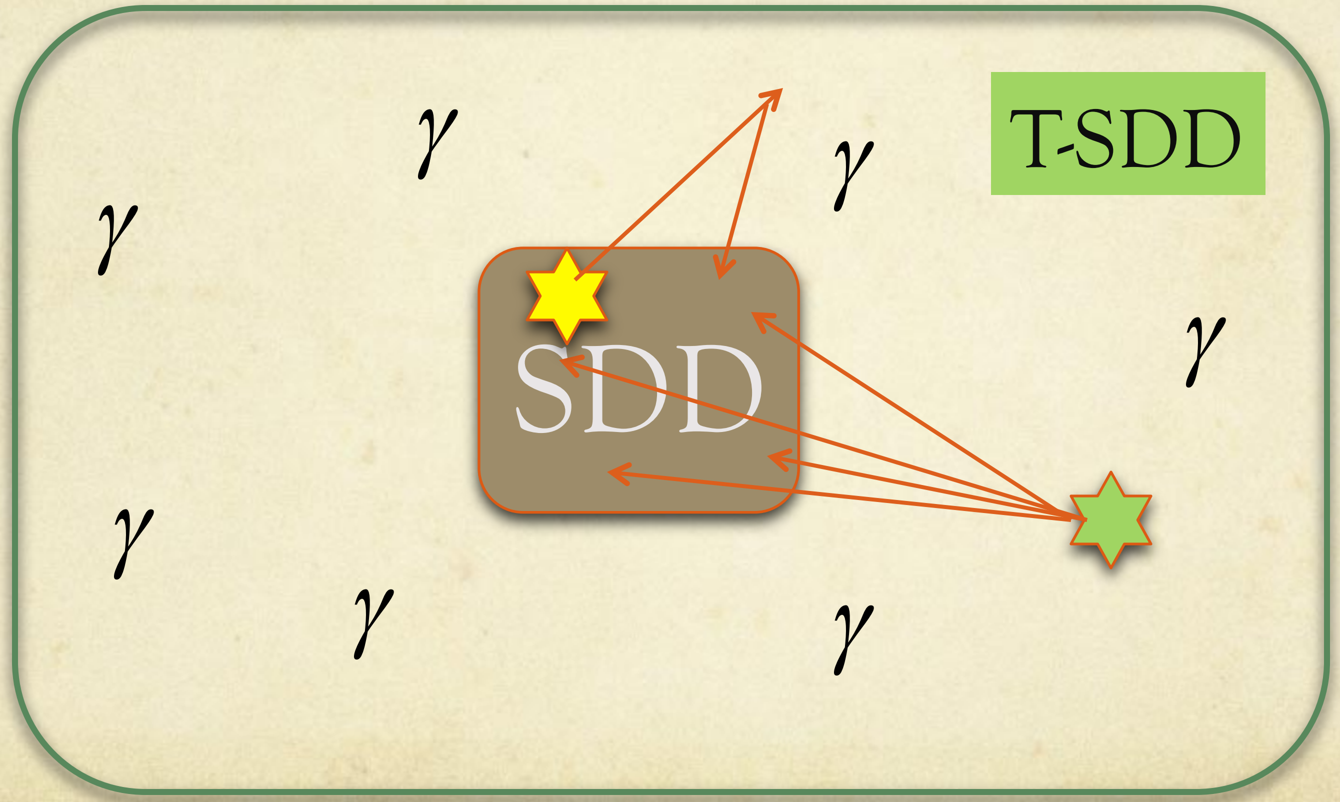
$$I_\gamma = (\gamma_1(A), \gamma_2(A)),$$

$$0 \le \gamma_1(A) = \max_{i \in S} \frac{r_i^{\bar{S}}(A)}{|a_{ii}| - r_i^S(A)}, \qquad \gamma_2(A) = \min_{j \in \bar{S}} \frac{|a_{jj}| - r_j^{\bar{S}}(A)}{r_j^S(A)}.$$

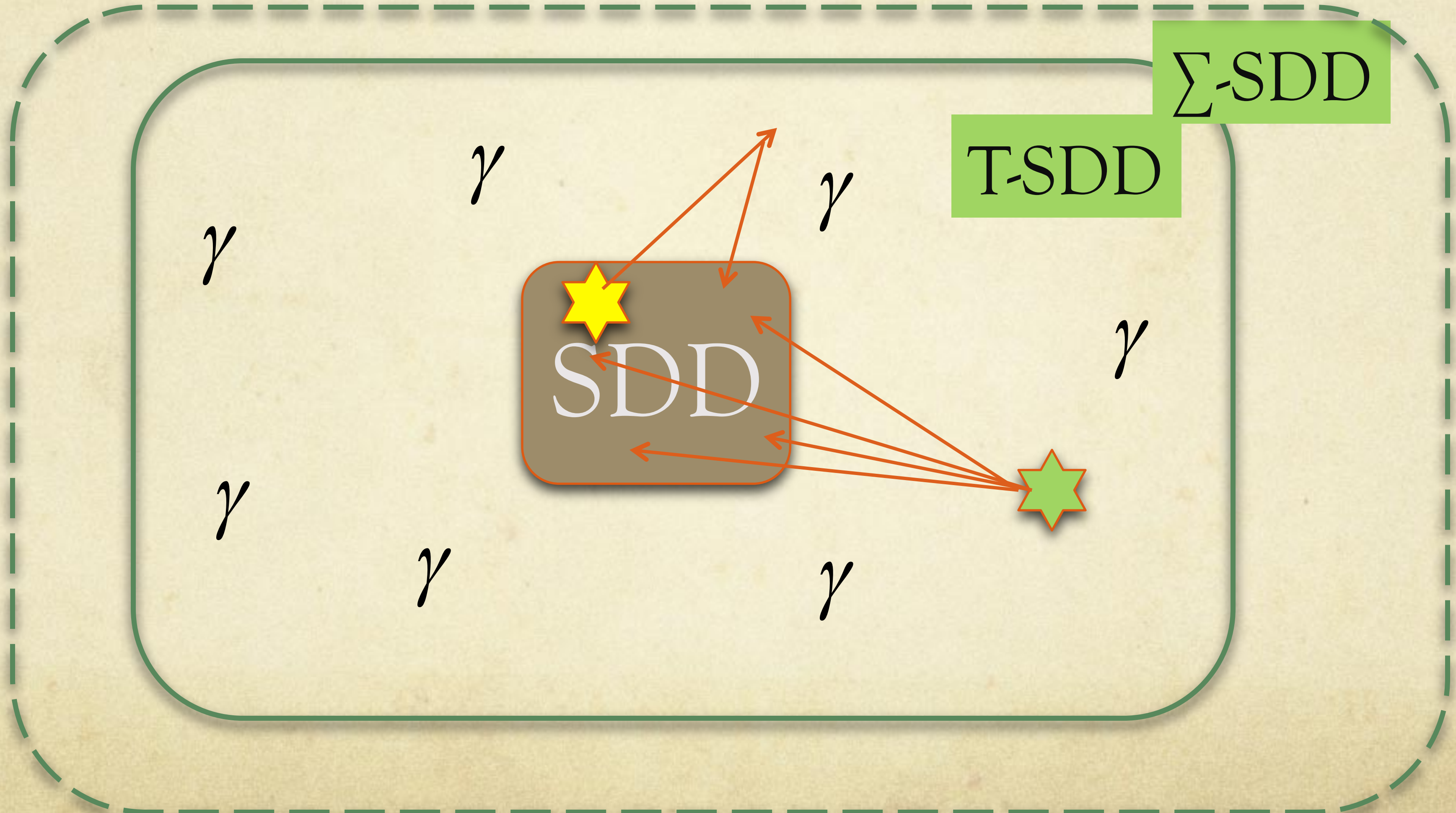
Diagonal scaling characterization & Scaling matrices



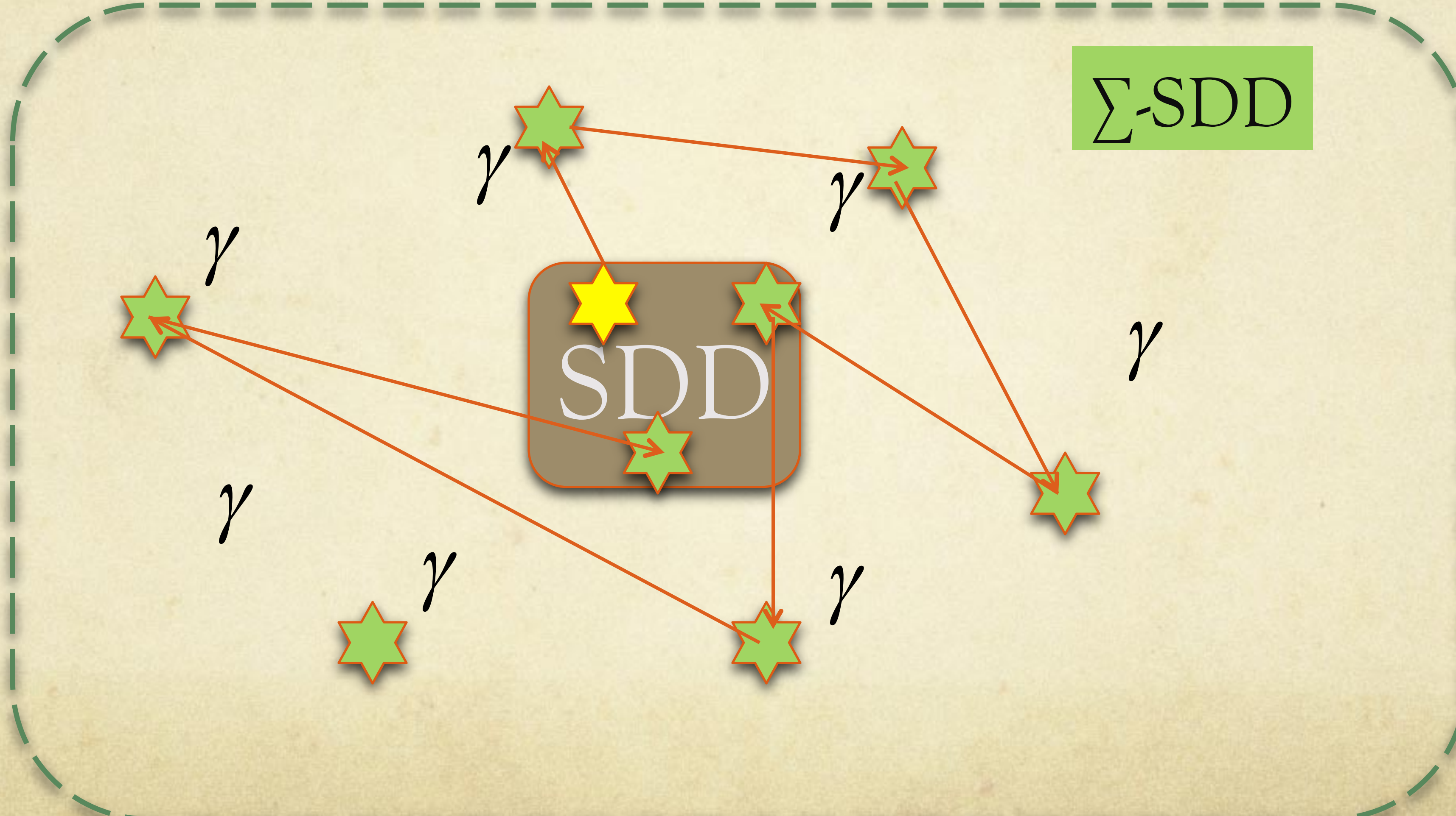
Diagonal scaling characterization & Scaling matrices



Diagonal scaling characterization & Scaling matrices



Diagonal scaling characterization & Scaling matrices



Eigenvalue localization

$$\Gamma_i^S(A) = \{z \in C : |z - a_{ii}| \le r_i^S(A)\}, \qquad i \in S,$$

$$V_{ij}^S(A) = \{z \in C : (|z - a_{ii}| - r_i^S(A))(|z - a_{jj}| - r_j^{\bar{S}}(A)) \le r_i^{\bar{S}}(A) r_j^S(A)\},$$

$i \in S, j \in \bar{S}.$

$$\sigma(A) \subseteq C^S(A) = \left(\bigcup_{i \in S} \Gamma_i^S(A)\right) \bigcup \left(\bigcup_{i \in S,\, j \in \bar{S}} V_{ij}^S(A)\right).$$

Cvetković, L., Kostić, V., Varga R.S.: A new Geršgorin-type eigenvalue inclusion set ETNA, 2004.

Varga R.S.: Geršgorin and his circles, Springer, Berlin, 2004.

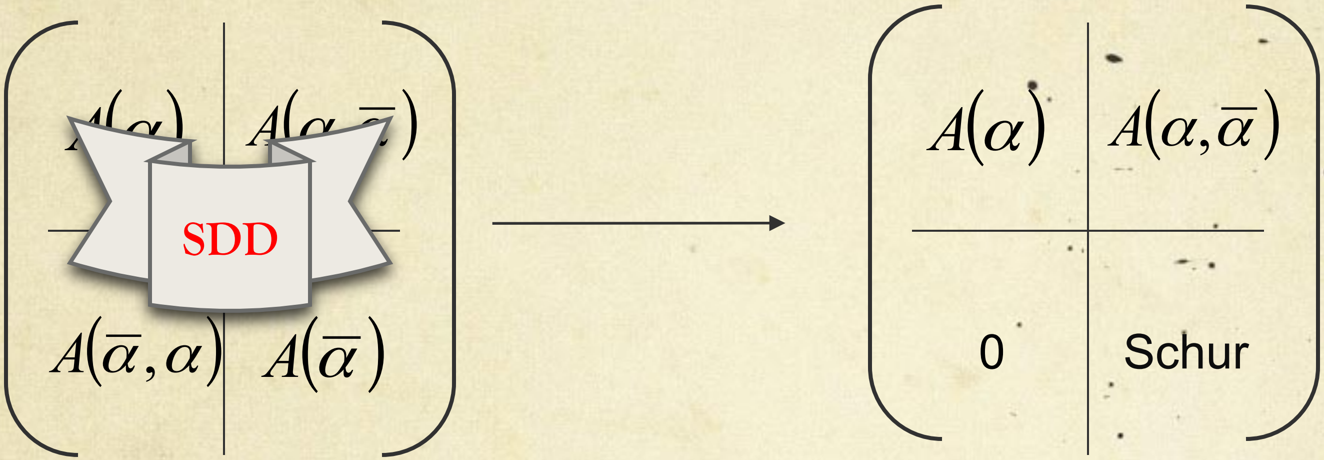
Schur complement

$$\left(\begin{array}{c|c} A(\alpha) & A(\alpha,\overline{\alpha}) \\ \hline A(\overline{\alpha},\alpha) & A(\overline{\alpha}) \end{array}\right) \longrightarrow \left(\begin{array}{c|c} A(\alpha) & A(\alpha,\overline{\alpha}) \\ \hline 0 & \textbf{Schur} \end{array}\right)$$

The Schur complement of a complex nxn matrix A, with respect to a proper subset α of index set N={1, 2,…, n}, is denoted by A/ α and defined to be:

$$A(\overline{\alpha}) - A(\overline{\alpha},\alpha)\left(A(\alpha)\right)^{-1} A(\alpha,\overline{\alpha})$$

Schur complement



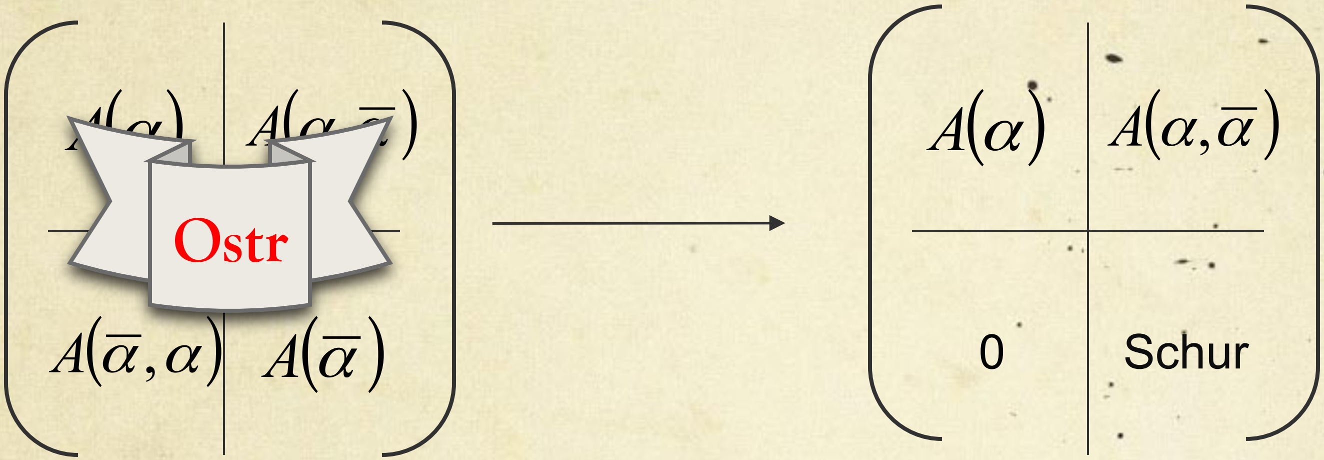
Carlson, D., Markham, T. : Schur complements of diagonally dominant matrices. Czech. Math. J. 29 (104) (1979), 246-251.

Schur complement

$$\left(\begin{array}{c|c} A(\alpha) & A(\alpha,\overline{\alpha}) \\ \hline A(\overline{\alpha},\alpha) & A(\overline{\alpha}) \end{array}\right) \longrightarrow \left(\begin{array}{c|c} A(\alpha) & A(\alpha,\overline{\alpha}) \\ \hline 0 & \text{SDD} \end{array}\right)$$

Carlson, D., Markham, T. : Schur complements of diagonally dominant matrices. Czech. Math. J. 29 (104) (1979), 246-251.

Schur complement



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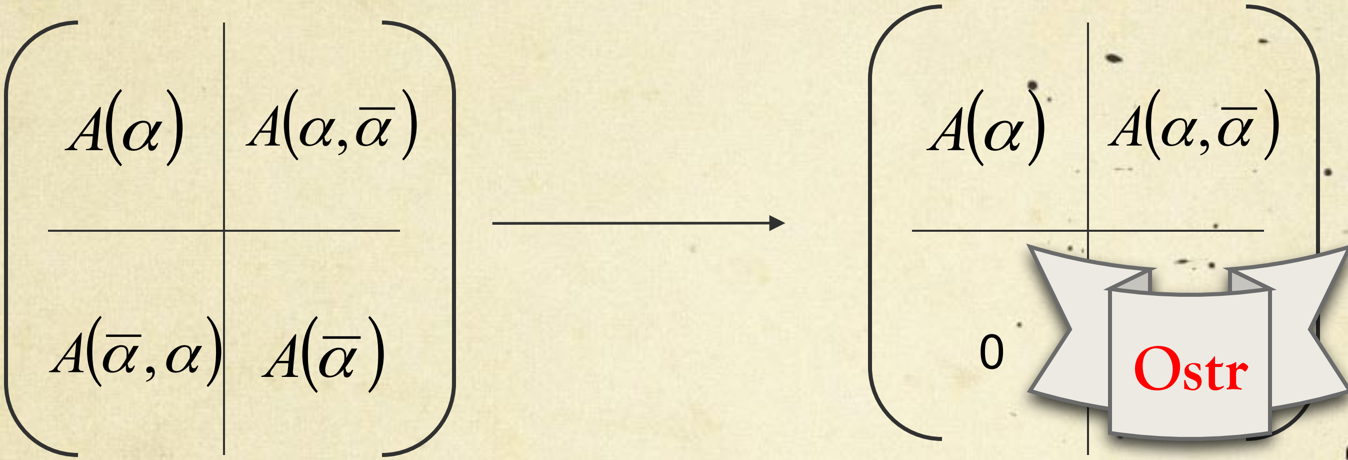
Li, B., Tsatsomeros, M.J. : Doubly diagonally dominant matrices. LAA 261 (1997)

Schur complement



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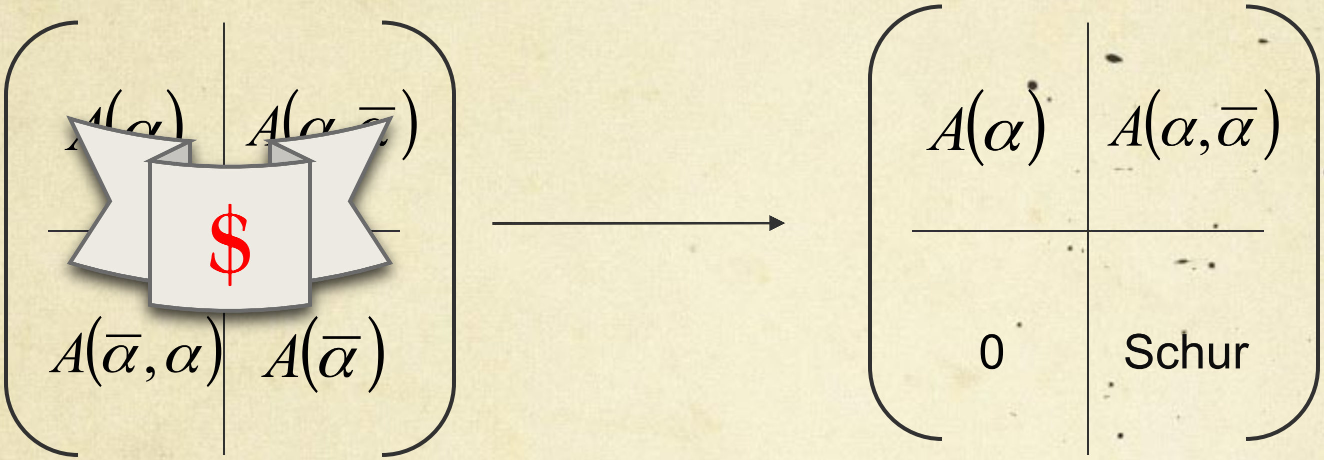

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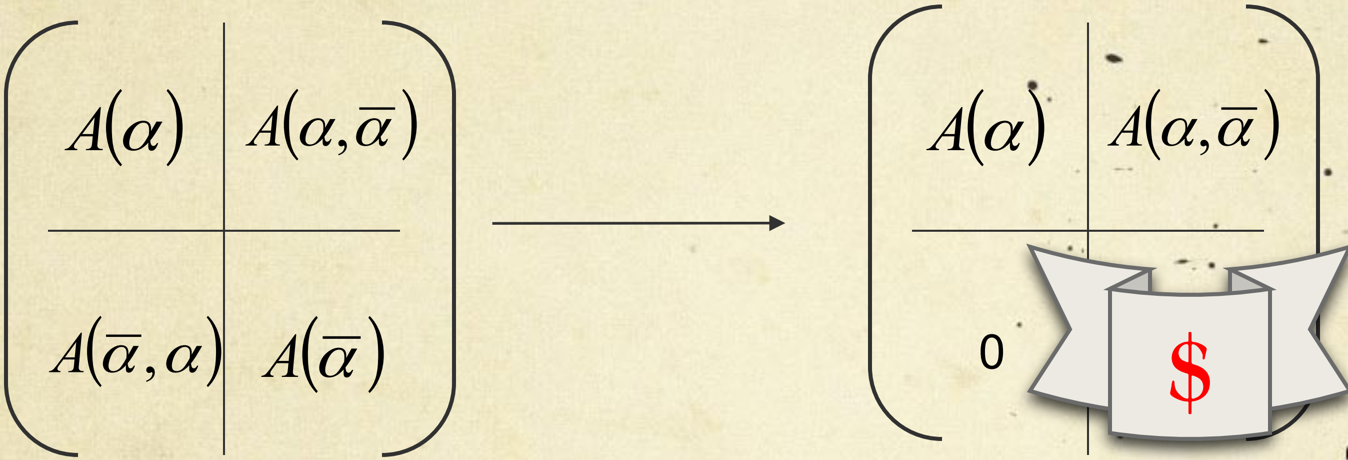
Zhang, F. : The Schur complement and its applications, Springer, NY, (2005).

Schur complement



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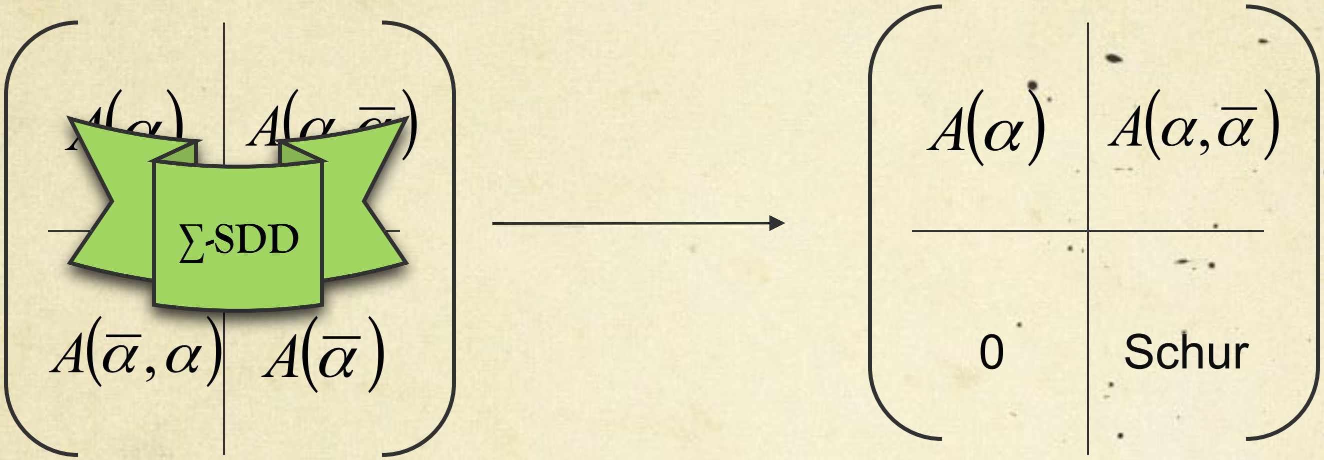
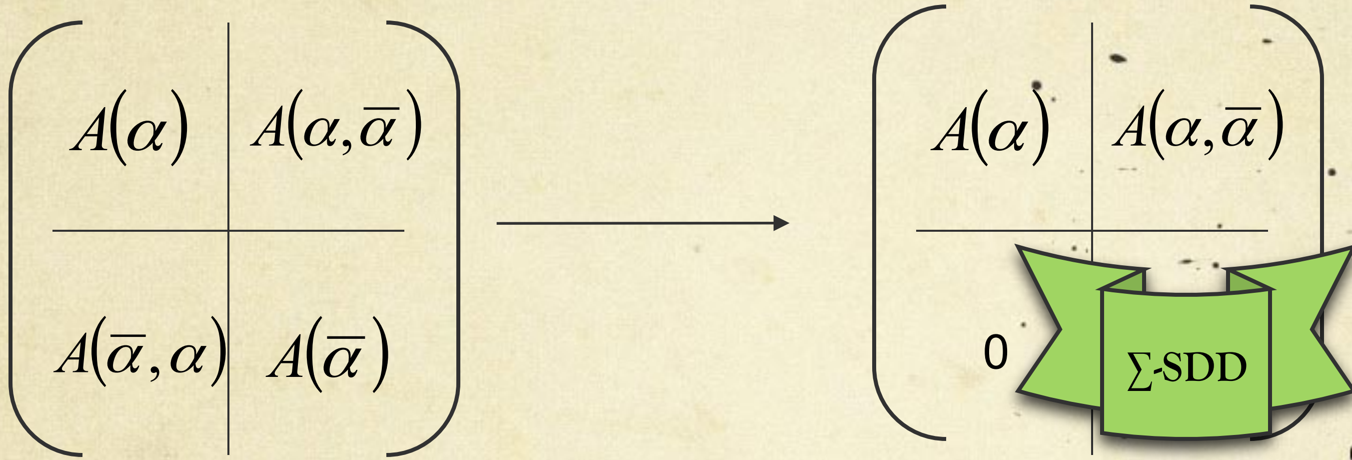
Schur complements of S-SDD



Cvetković, Lj., Kostić, V., Kovačević, M., Szulc, T. : Further results on H-matrices and their Schur complements. AMC (2008)

Liu, J., Huang, Y., Zhang, F. : The Schur complements of generalized doubly diagonally dominant matrices. LAA (2004)

Schur complements of S-SDD



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Liu, J., Huang, Y., Zhang, F. : The Schur complements of generalized doubly diagonally dominant matrices. LAA (2004)

Schur complements of S-SDD

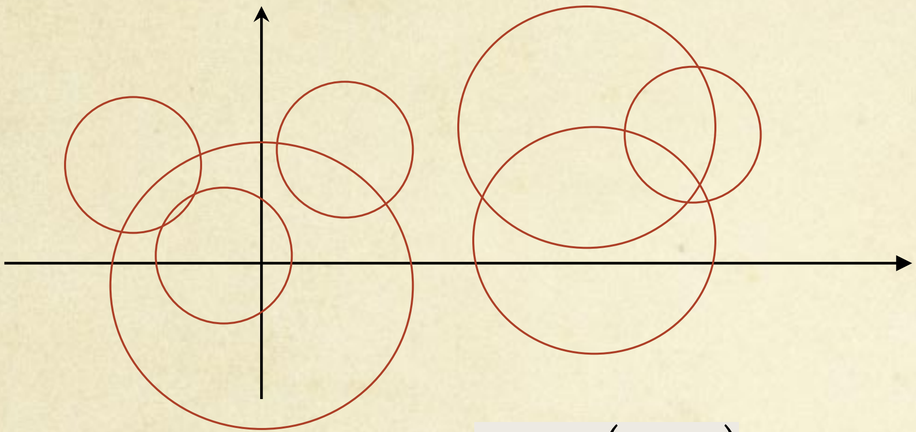
Theorem1. Let $A=[a_{ij}]_{n\times n}$ be an ∑-SDD matrix. Then for any nonempty proper subset α of N, A/ α is also an ∑-SDD matrix. More precisely, if A is an S-SDD matrix, then A/ α is an (S\ α)-SDD matrix.

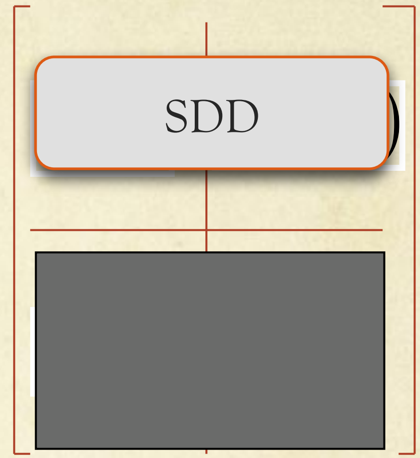
Theorem2. Let $A=[a_{ij}]_{n\times n}$ be an S-SDD matrix. Then for any nonempty proper subset α of N such that S is a subset of α or N\S is a subset of α, A/ α is an SDD matrix.

Cvetković, Lj., Kostić, V., Kovačević, M., Szulc, T. : Further results on H-matrices and their Schur complements. AMC (2008)

Cvetković, Lj., Nedović, M. : Special H-matrices and their Schur and diagonal-Schur complements. AMC (2009)

Eigenvalues of the SC





$$\lambda \in \sigma(A/\alpha)$$

$$\sigma(A) \subseteq \Gamma(A) = \bigcup_{i \in N} \Gamma_i(A) = \bigcup_{i \in N} \left\{ z \in C : |z - a_{ii}| \leq r_i(A) = \sum_{j \in N \setminus \{i\}} |a_{ij}| \right\}$$

Liu, J., Huang, Z., Zhang, J. : The dominant degree and disc theorem for the Schur complement. AMC (2010)

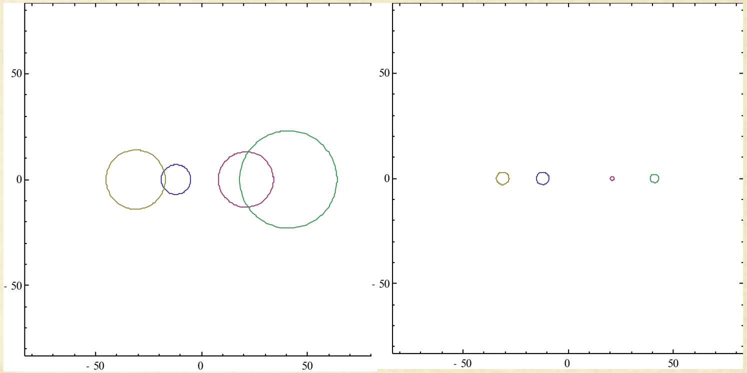
Eigenvalues of the SC of S-SDD

$$A \in S - SDD, \qquad S = \alpha.$$

$$\gamma > \max_{i \in \alpha} \frac{r_i^{\overline{\alpha}}(A)}{|a_{ii}| - r_i^{\alpha}(A)},$$

$$\sigma(A/\alpha) = \sigma\left(\left(W^{-1}AW\right)/\alpha\right) \subseteq \bigcup_{j \in \overline{\alpha}} \Gamma_j\left(W^{-1}AW\right),$$

$$R_j = \gamma r_j^{\alpha}(A) + r_j^{\overline{\alpha}}(A).$$



Weighted Geršgorin set for the Schur complement matrix

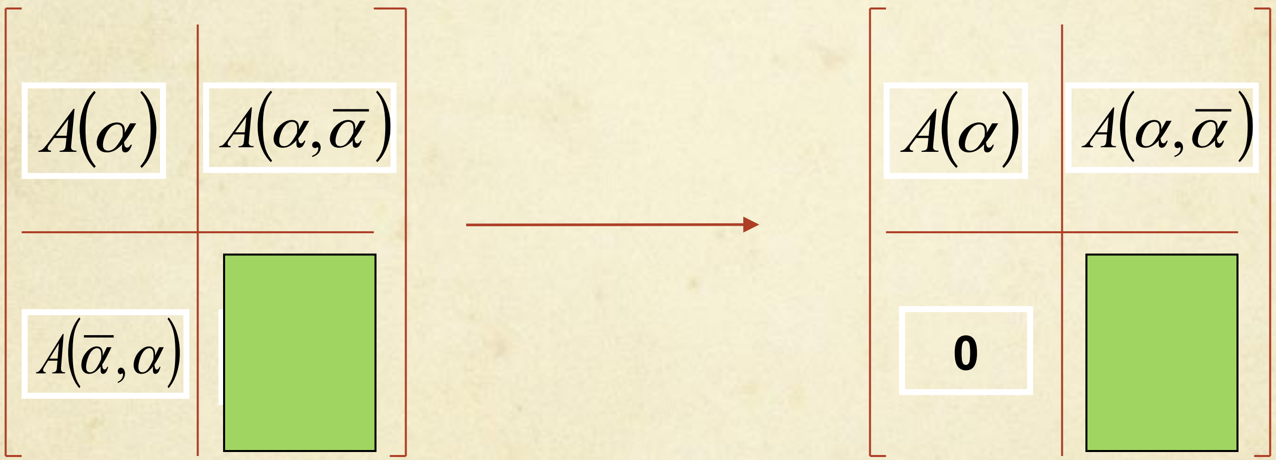
Cvetković, Lj., Nedović, M. : Eigenvalue localization refinements for the Schur complement. AMC (2012)

Cvetković, Lj., Nedović, M. : Diagonal scaling in eigenvalue localization for the Schur complement. PAMM (2013)

Eigenvalues of the SC

- Let A be an SDD matrix with real diagonal entries and let α be a proper subset of N. Then, A/α and A(N\α) have the same number of eigenvalues whose real parts are greater (less) than w (resp. $-w$), where

$$w(A) = \min_{j \in \overline{\alpha}} \left[|a_{jj}| - r_j(A) + \min_{i \in \alpha} \frac{|a_{ii}| - r_i(A)}{|a_{ii}|} r_j^{\alpha}(A) \right]$$



Liu, J., Huang, Z., Zhang, J. : The dominant degree and disc theorem for the Schur complement. AMC (2010)

Eigenvalues of the SC of S-SDD

- Let A be an S-SDD matrix with real diagonal entries and let α be a proper subset of N. Then, A/ α and A(N\α) have the same number of eigenvalues whose real parts are greater (less) than w (resp. -w), where

$$w = w\left(W^{-1} A W\right)$$

Remarks:

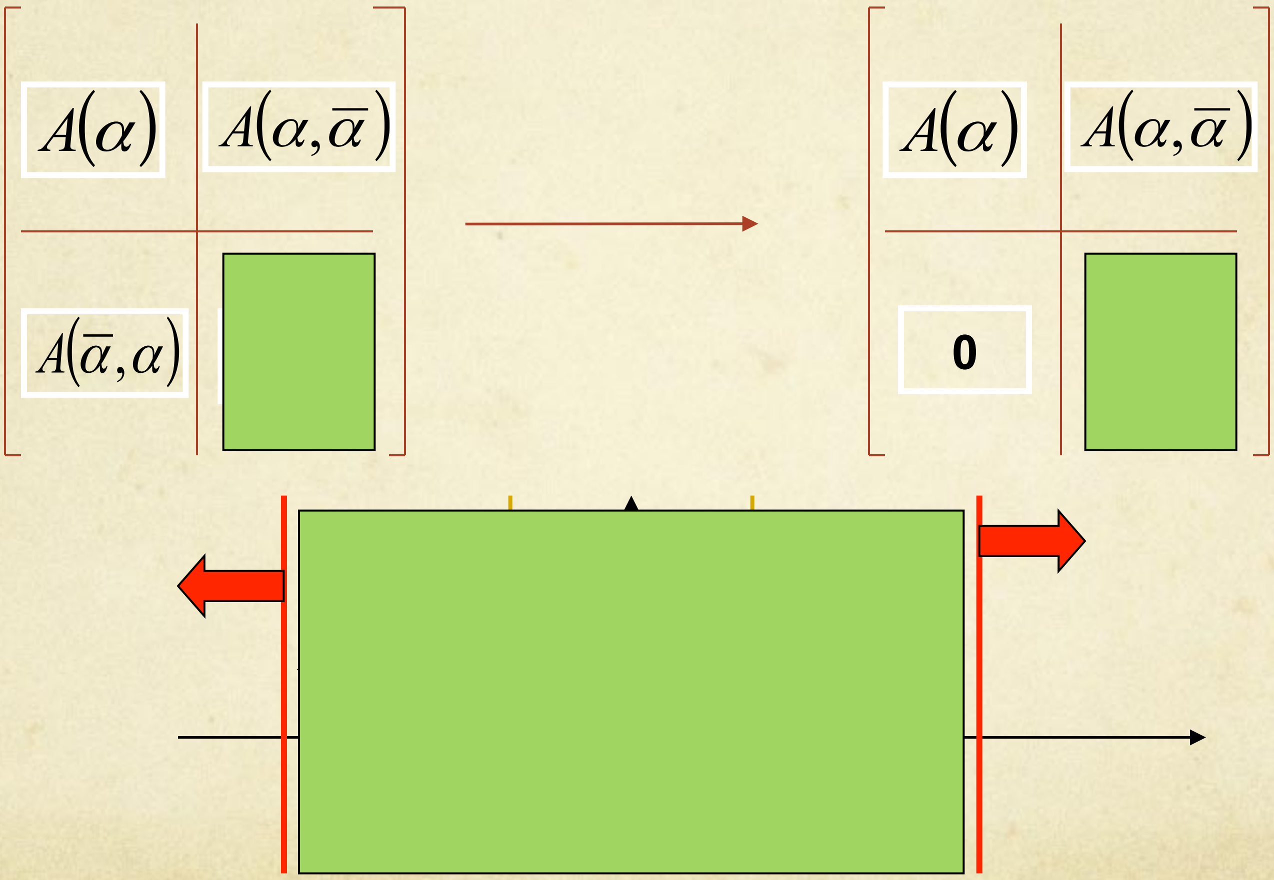
•This result covers a wider class of matrices.

•By changing the parameter in the scaling matrix we obtain more vertical bounds with the same separating property.

•We can apply it to an SDD matrix, observing that it belongs to T-SDD class for any T subset of N.

Cvetković, Lj., Nedović, M. : Eigenvalue localization refinements for the Schur complement. AMC (2012)

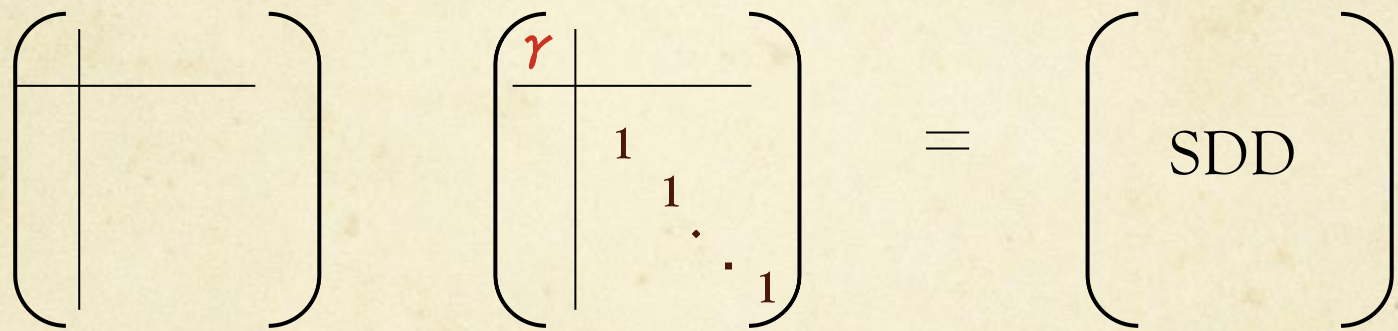
Eigenvalues of the SC of S-SDD



Dashnic-Zusmanovich

A matrix $A=[a_{ij}]_{n\times n}$ is a Dashnic-Zusmanovich (DZ) matrix if there exists an index *i* in N such that

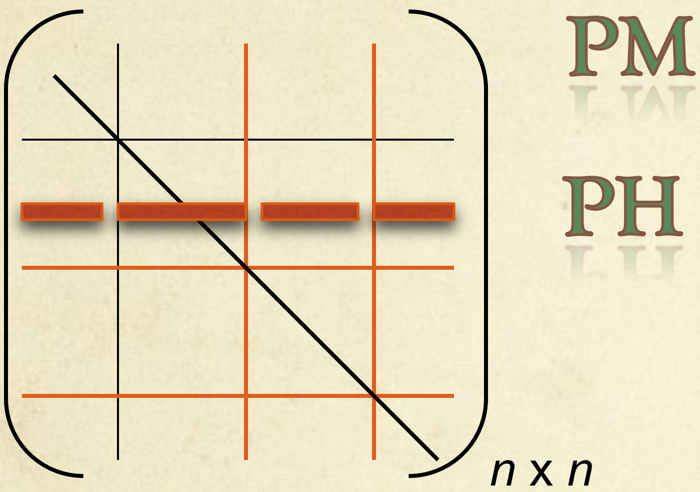
$$|a_{ii}|\left(|a_{jj}| - r_j(A) + |a_{ji}|\right) > r_i(A)|a_{ji}|, \qquad j \neq i, j \in N.$$



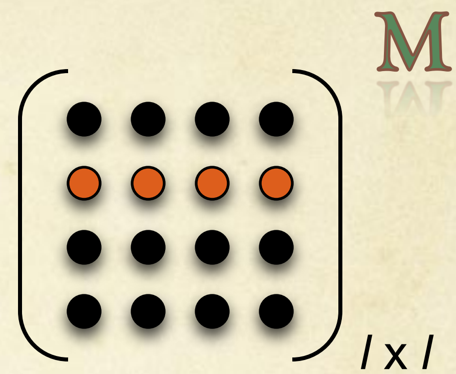
Dashnic, L. S., Zusmanovich, M.S.: O nekotoryh kriteriyah regulyarnosti matric i lokalizacii spectra. Zh. vychisl. matem. i matem. fiz. (1970)

Dashnic, L. S., Zusmanovich, M.S.: K voprosu o lokalizacii harakteristicheskih chisel matricy. Zh. vychisl. matem. i matem. fiz. (1970)

PH-matrices



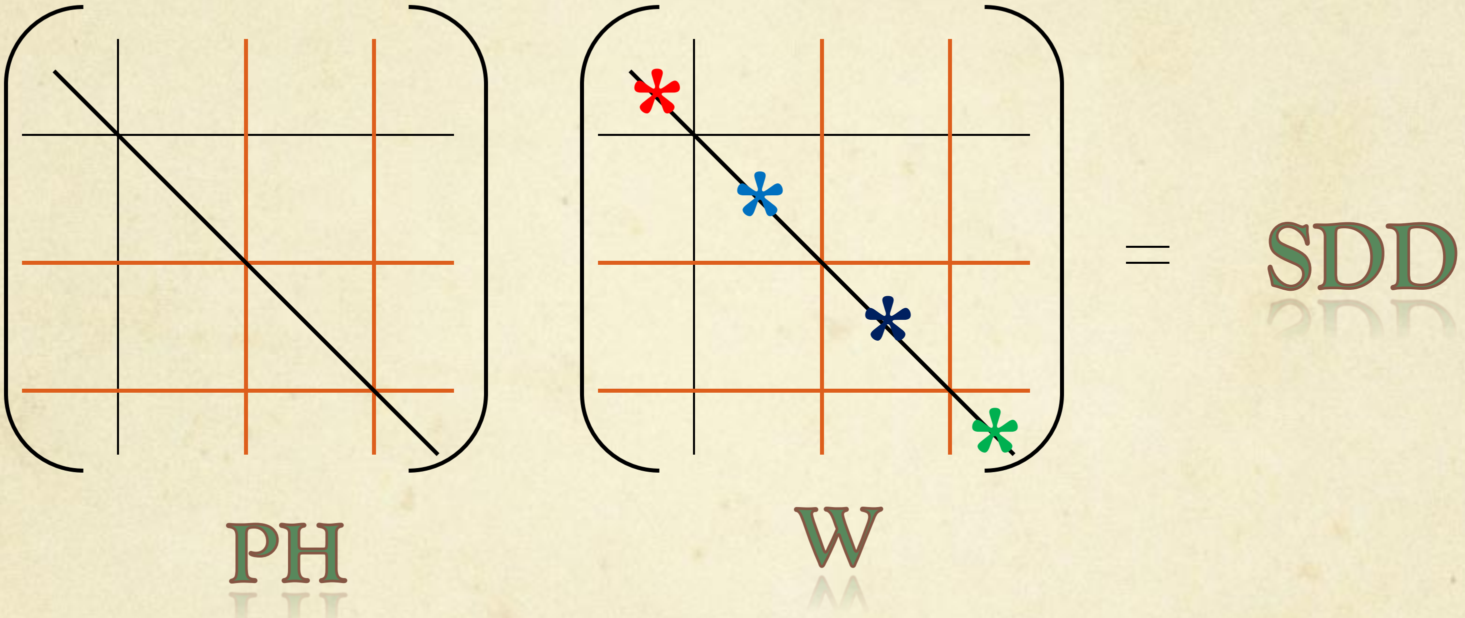
$$\pi : S_1, S_2, \ldots, S_l$$



Aggregated matrices

Kolotilina, L. Yu. : Diagonal dominance characterization of PM- and PH-matrices. Journal of Mathematical Sciences (2010)

PH-matrices



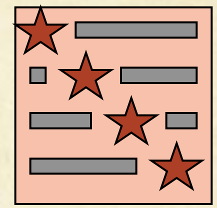
Kolotilina, L. Yu. : Diagonal dominance characterization of PM- and PH-matrices. Journal of Mathematical Sciences (2010)

Nekrasov matrices

- A complex matrix $A=[a_{ij}]_{nxn}$ is SDD-matrix if for each *i* from N it holds that

$$|a_{ii}| > r_i(A) = \sum_{j\in N,\, j\neq i} |a_{ij}|$$

$$d(A) > r(A)$$

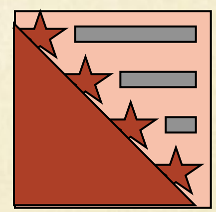


- A complex matrix $A=[a_{ij}]_{nxn}$ is a Nekrasov-matrix if for each *i* from N it holds that

$$|a_{ii}| > h_i(A), \qquad h_1(A) = r_1(A),$$

$$h_i(A) = \sum_{j=1}^{i-1} |a_{ij}| \frac{h_j(A)}{|a_{jj}|} + \sum_{j=i+1}^{n} |a_{ij}|, \qquad i = 2,3,\ldots,n.$$

$$d(A) > h(A)$$



$$A = D - L - U$$

Nekrasov matrices

A complex matrix A=[aij]nxn is a Nekrasov-matrix if for each *i* from N it holds that

$$|a_{ii}| > h_i(A), \qquad h_1(A) = r_1(A),$$

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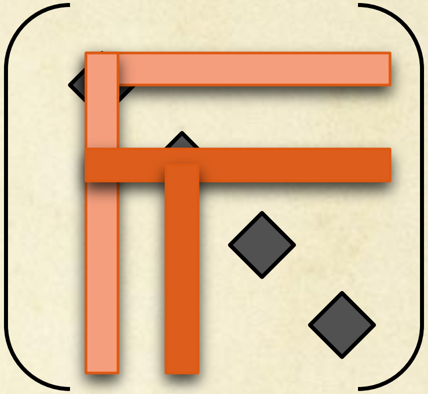
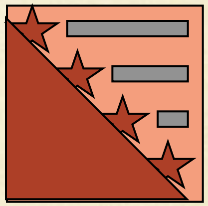
$$i = 2,3,\ldots,n.$$



Nekrasov row sums



$$d(A) > h(A)$$

$$A = D - L - U$$

Nekrasov matrices and scaling

Theorem. Let $A=[a_{ij}]_{nxn}$ be a Nekrasov matrix with nonzero Nekrasov row sums. Then, for a diagonal positive matrix D where

$$d_i = \varepsilon_i \frac{h_i(A)}{|a_{ii}|}, \qquad i = 1,\ldots,n,$$

and $(\varepsilon_i)_{i=1}^n$ is an increasing sequence of numbers with

$$\varepsilon_1 = 1, \qquad \varepsilon_i \in \left(1, \frac{|a_{ii}|}{h_i(A)}\right), \qquad i = 2,\ldots,n,$$

the matrix AD is an SDD matrix.

Szulc, T., Cvetković, Lj., Nedović, M. : Scaling technique for Nekrasov matrices. AMC (2015) (in print)

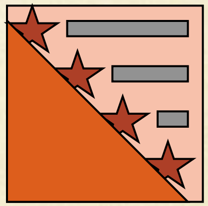
Nekrasov matrices and permutations

- Unlike SDD and H, Nekrasov class is NOT closed under similarity (simultaneous) permutations of rows and columns!
- Given a permutation matrix P, a complex matrix $A=[a_{ij}]_{n\times n}$ is called

P-Nekrasov if

$$\left|\left(P^T AP\right)_{ii}\right| > h_i\left(P^T AP\right), \qquad i \in N,$$

$$d\left(P^T AP\right) > h\left(P^T AP\right).$$



- The union of all P-Nekrasov=

Gudkov class

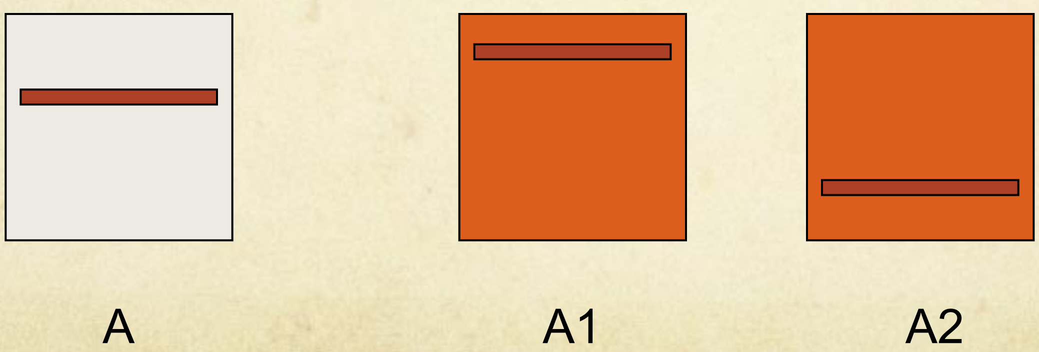
$$A = D - L - U$$

{P1,P2} – Nekrasov matrices

- Suppose that for the given matrix $A=[a_{ij}]_{n\times n}$ and two given permutation matrices P1 and P2

$$d(A) > \min\{h^{P_1}(A), h^{P_2}(A)\}, \qquad h^{P_k}(A) = P_k h(P_k^T A P_k), \qquad k = 1,2.$$

We call such a matrix {P1,P2} – Nekrasov matrix.



{P1,P2} – Nekrasov matrices

- Theorem1. Every {P1, P2} – Nekrasov matrix is nonsingular.
- Theorem2. Every {P1, P2} – Nekrasov matrix is an H – matrix.
- Theorem3. Given an arbitrary set of permutation matrices

$$\Pi_n = \{P_k\}_{k=1}^{p}$$

every Π_n – Nekrasov matrix is nonsingular, moreover, it is an H – matrix.

Cvetković, Lj., Kostić, V., Nedović, M. : Generalizations of Nekrasov matrices and applications. (2014)

Max-norm bounds for the inverse of {P1,P2} – Nekrasov matrices

- Theorem1. Suppose that for a given set of permutation matrices {P1, P2}, a complex matrix $A=[a_{ij}]_{n\times n}$, n>1, is a {P1, P2} – Nekrasov matrix. Then,

$$\|A^{-1}\|_\infty \le \frac{\max_{i\in N}\left[\min\left\{\frac{z_i^{P_1}(A)}{|a_{ii}|}, \frac{z_i^{P_2}(A)}{|a_{ii}|}\right\}\right]}{\min_{i\in N}\left[1-\min\left\{\frac{h_i^{P_1}(A)}{|a_{ii}|}, \frac{h_i^{P_2}(A)}{|a_{ii}|}\right\}\right]},$$

where

$$z_1(A)=r_1(A), \qquad z_i(A)=\sum_{j=1}^{i-1}|a_{ij}|\frac{z_j(A)}{|a_{jj}|}+1, \qquad i=2,3,\dots,n,$$

$$z(A)=[z_1(A), \quad \dots, \quad z_n(A)]^T, \qquad z^P(A)=Pz(P^TAP).$$

Cvetković, Lj., Kostić, V., Nedović, M. : Generalizations of Nekrasov matrices and applications. (2014)

Max-norm bounds for the inverse of {P1,P2} – Nekrasov matrices

- Theorem2. Suppose that for a given set of permutation matrices {P1, P2}, a complex matrix A=[aij]nxn, n>1, is a {P1, P2} – Nekrasov matrix. Then,

$$\|A^{-1}\|_\infty \leq \frac{\max_{i\in N}\left[\min\left\{z_i^{P_1}(A), z_i^{P_2}(A)\right\}\right]}{\min_{i\in N}\left[|a_{ii}| - \min\left\{h_i^{P_1}(A), h_i^{P_2}(A)\right\}\right]},$$

$$z_1(A) = r_1(A), \qquad z_i(A) = \sum_{j=1}^{i-1} |a_{ij}| \frac{z_j(A)}{|a_{jj}|} + 1, \qquad i = 2,3,\ldots,n,$$

$$z(A) = [z_1(A), \quad \ldots, \quad z_n(A)]^T, \qquad z^P(A) = Pz(P^T AP).$$

Cvetković, Lj., Kostić, V., Nedović, M. : Generalizations of Nekrasov matrices and applications. (2014)

Numerical examples

- Observe the given matrix B and permutation matrices P1 and P2.

$$B = \begin{bmatrix} 60 & -15 & -15 & -15 \\ -75 & 105 & -45 & 0 \\ -60 & -60 & 120 & -15 \\ -15 & -15 & -15 & 45 \end{bmatrix}, \quad P_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad P_2 = I,$$

$$h_1^{P_1}(B) = 45, \quad h_2^{P_1}(B) = 101.25, \quad h_3^{P_1}(B) = 116.607, \quad h_4^{P_1}(B) = 41.25,$$
$$h_1^{P_2}(B) = 45, \quad h_2^{P_2}(B) = 101.25, \quad h_3^{P_2}(B) = 117.857, \quad h_4^{P_2}(B) = 40.4464.$$

- Notice that B is a Nekrasov matrix.

Cvetković, Lj., Kostić, V., Doroslovački, K. : Max-norm bounds for the inverse of S-Nekrasov matrices. (2012)

Numerical examples

- Observe the given matrix B and permutation matrices P1 and P2.

$$B = \begin{bmatrix} 60 & -15 & -15 & -15 \\ -75 & 105 & -45 & 0 \\ -60 & -60 & 120 & -15 \\ -15 & -15 & -15 & 45 \end{bmatrix}, \quad P_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad P_2 = I,$$

$$\left\| B^{-1} \right\|_\infty \leq 1.76842$$

$$\left\| B^{-1} \right\|_\infty \leq 0.968421$$

$$\left\| B^{-1} \right\|_\infty = 0.6843.$$

Although the given matrix IS a Nekrasov matrix, in this way we obtained a better bound for the norm of the inverse.

∑– Nekrasov matrices

- Given any matrix A and any nonempty proper subset S of N we say that A is an S-Nekrasov matrix if

$$|a_{ii}| > h_i^S(A), i \in S,$$

$$|a_{jj}| > h_j^{\bar{S}}(A), j \in \bar{S},$$

$$h_1^S(A) = r_1^S(A)$$

$$h_i^S(A) = \sum_{j=1}^{i-1} |a_{ij}| \frac{h_j^S(A)}{|a_{jj}|} + \sum_{j=i+1, j\in S}^{n} |a_{ij}|$$

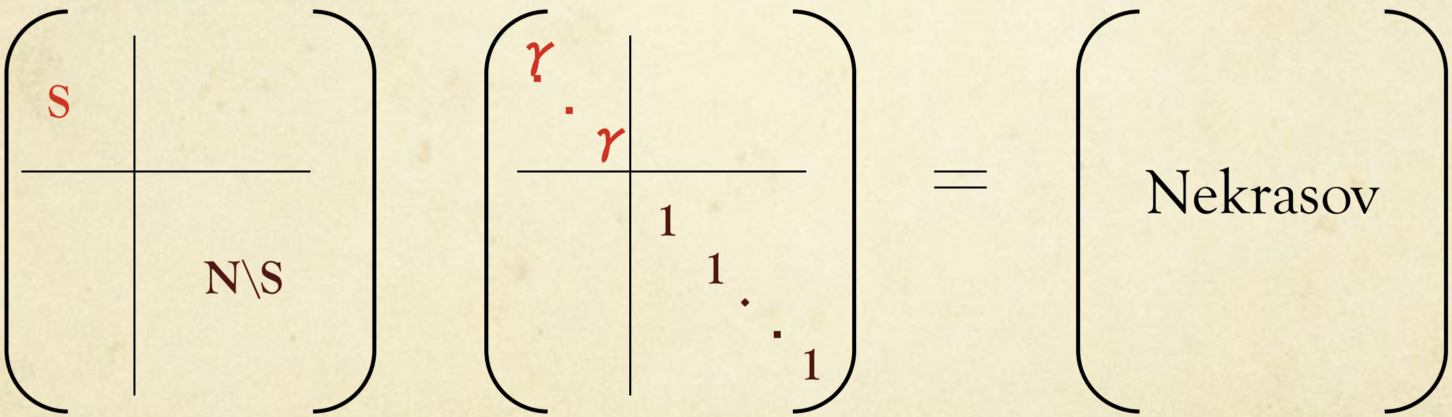
$$\left(|a_{ii}| - h_i^S(A)\right)\left(|a_{jj}| - h_j^{\bar{S}}(A)\right) > h_i^{\bar{S}}(A) h_j^S(A), \qquad i \in S, j \in \bar{S}.$$

- If there exists a nonempty proper subset S of N such that A is an S-Nekrasov matrix, then we say that A belongs to the class of ∑-Nekrasov matrices.

∑– Nekrasov matrices

Cvetković, Lj., Kostić, V., Rauški, S. : A new subclass of H-matrices. AMC (2009)

$$\boldsymbol{W}^{S} = \left\{ W = diag(w_1, w_2, \ldots, w_n) : w_i = \gamma > 0 \quad for \quad i \in S \quad and \quad w_i = 1 \quad for \quad i \in \overline{S} \right\}$$



∑– Nekrasov matrices

Cvetković, Lj., Kostić, V., Rauški, S. : A new subclass of H-matrices. AMC (2009)

Cvetković, Lj., Nedović, M. : Special H-matrices and their Schur and diagonal-Schur complements. AMC (2009)

Szulc, T., Cvetković, Lj., Nedović, M. : Scaling technique for Nekrasov matrices. AMC (2015) (in print)

Nonstrict conditions

DD - matrices

IDD-matrices

$$|a_{ii}| \geq r_i(A), \qquad i = 1,2,\ldots,n.$$

$|a_{kk}| > r_k(A)$ for one k in N

irreducibility

Olga Taussky (1948)

CDD-matrices

$$|a_{ii}| \geq r_i(A), \qquad i = 1,2,\ldots,n.$$

$|a_{kk}| > r_k(A)$ for one k in N

non-zero chains

T. Szulc (1995)

Nonstrict conditions

DD - matrices

IDD-matrices	CDD-matrices
$\lvert a_{ii} \rvert \geq r_i(A), \quad i = 1,2,\ldots,n.$	$\lvert a_{ii} \rvert \geq r_i(A), \quad i = 1,2,\ldots,n.$
$\lvert a_{kk} \rvert > r_k(A)$ for one k in N	$\lvert a_{kk} \rvert > r_k(A)$ for one k in N
irreducibility	non-zero chains
Olga Taussky (1948)	T. Szulc (1995)

Li, W. : On Nekrasov matrices. LAA (1998)

Nonstrict conditions

S-IDD

Given an irreducible complex matrix $A=[a_{ij}]_{n\times n}$, if there is a nonempty proper subset S of N such that the following conditions hold, where the last inequality becomes strict for at least one pair of indices *i* in S and *j* in N\S, then A is an H-matrix.

$$|a_{ii}| \geq r_i^S(A) = \sum_{j\in S, j\neq i} |a_{ij}|, \qquad i \in S$$

$$\left(|a_{ii}| - r_i^S(A)\right)\left(|a_{jj}| - r_j^{\bar{S}}(A)\right) \geq r_i^{\bar{S}}(A) r_j^S(A), \qquad i \in S, j \in \bar{S}.$$

Cvetković, Lj., Kostić, V. : New criteria for identifying H-matrices. JCAM (2005)

Nonstrict conditions

S-CDD

Given a complex matrix $A=[a_{ij}]_{n\times n}$, if there is a nonempty proper subset S of N such that the following conditions hold, where the last inequality becomes strict for at least one pair of indices *i* in S and *j* in N\S, and for every pair of indices *i* in S and *j* in N\S for which equality holds there exists a pair of indices *k* in S and *l* in N\S for which strict inequality holds and there is a path from *i* to *l* and from *j* to *k*, then A is an H-matrix.

$$|a_{ii}| \geq r_i^S(A) = \sum_{j\in S, j\neq i} |a_{ij}|, \qquad i\in S$$

$$\left(|a_{ii}| - r_i^S(A)\right)\left(|a_{jj}| - r_j^{\bar{S}}(A)\right) \geq r_i^{\bar{S}}(A) r_j^S(A), \qquad i\in S, j\in \bar{S}.$$

Cvetković, Lj., Kostić, V. : New criteria for identifying H-matrices. JCAM (2005)

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Cvetković, Lj., Kostić, V., Nedović, M. : Generalizations of Nekrasov matrices and applications. (2014)

Szulc, T., Cvetković, Lj., Nedović, M. : Scaling technique for Nekrasov matrices. AMC (2015) (in print)

Mat Triad
Coimbra 2015

THANK YOU FOR YOUR ATTENTION!

HVALA NA PAŽNJI!