# The finite basis problem for Kauffman monoids

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Kauffman monoids $\mathcal{K}_n$, $n \geq 2$, are monoids whose linear spans are the famous Temperley–Lieb algebras [6], which first arose in statistical mechanics but then turned out to play a prominent role in several 'fashionable' parts of mathematics such as knot theory and low-dimensional topology (see [3]), topological quantum field theory, quantum groups etc. In the talk we shall recall the original 'pictorial' definition of Kauffman monoids but for this abstract one can define $\mathcal{K}_n$ as the monoid with $n$ generators $c, h_1, \ldots, h_{n-1}$ subject to the relations

$$\begin{aligned} h_i h_j &= h_j h_i \quad \text{if } |i-j| \geq 2, \\ h_i h_j h_i &= h_i \quad \text{if } |i-j| = 1, \\ h_i h_i &= c h_i, \\ c h_i &= h_i c. \end{aligned}$$

The name "Kauffman monoids" has been suggested in [2]; another name in use is "Temperley–Lieb–Kauffman monoids", see [1]. The semigroup structure of the Kauffman monoids has been deeply studied in [4]. Using structural results of [4] and the techniques from [5], we prove here

**Theorem.** *The identities of the Kauffman monoid $\mathcal{K}_n$, $n \geq 4$, are not finitely based.*

The monoid $\mathcal{K}_2$ is commutative, and thus, its identities are finitely based. The question of whether or not the identities of the monoid $\mathcal{K}_3$ are finitely based still remains open. Yet another open question is whether or not the monoids $\mathcal{K}_n$ have finitely based *unary semigroup* identities (the monoids are endowed with a fairly natural unary operation making them *-regular semigroups).

## REFERENCES

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