On the complexity of equivalence and isomorphism of primitive positive formulas

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A *primitive positive* formula is a first-order formula defined from atomic formulas and equality of variables using conjunction and existential quantification. In this talk, we study the complexity of two basic questions associated to primitive positive formulas:

(1) Given two such formulas having the same free variables and a relational structure, are the formulas equivalent over the structure? That is, do they admit the same satisfying assignments?
(2) Given two such formulas and a relational structure, are the formulas isomorphic over the structure? By isomorphic, we mean that there is a way to rename the free variables of one formula with the free variables of the other so that the formulas are equivalent.

We study both of these problems with respect to various fixed structures. That is, we parameterize each of these problems with respect to the structure to obtain a family of problems, containing one member for each structure, and study the resulting two families of problems.

As has been shown for the Constraint Satisfaction Problem, we demonstrate that to each structure one can associate an algebra in such a way that the complexity of all structures associated to the same algebra is the same. We present a number of sufficient conditions for hardness for various complexity classes, as well as sufficient conditions for containment in various complexity classes.

The presented work is joint with S. BOVA (Vanderbilt University, Nashville) and H. CHEN (University Pompeu Fabra, Barcelona).