E-solid locally inverse semigroups

MÁRIA B. SZENDREI
Bolyai Institute, University of SZEGED
M.Szendrei@math.u-szeged.hu

In the structure theory of regular semigroups, the subclasses $\mathcal{LI}$ and $\mathcal{ES}$ of all locally inverse and all E-solid semigroups, respectively, have been studied since the late 1970's. In the 1990's, exactly these two subclasses turned out to be the maximal subclasses of regular semigroups where a universal algebraic theory reminiscent to the theory of varieties can be developed.

The class of all inverse semigroups and the class of all completely simple semigroups are the two best known subclasses of $\mathcal{LI} \cap \mathcal{ES}$. It is a natural question to ask whether each E-solid locally inverse semigroup can be built up from inverse and completely simple semigroups. Our main result is a contribution to this topic.

Let S be a regular semigroup and denote its set of idempotents by $E(S)$. Then S is said to be *locally inverse* if eSe is an inverse monoid for every $e \in E(S)$. Moreover, S is called *E-solid* if its subsemigroup generated by $E(S)$ is completely regular. It is well known that each regular semigroup S possesses a least congruence γ such that S/γ is an inverse semigroup. Furthermore, S is E-solid if and only if each idempotent γ-class constitutes a completely simple subsemigroup in S.

In order to be able to formulate our main result, we need the notion of a *λ-semidirect product*. Without defining it formally, let us mention here only that it is a construction within the class $\mathcal{LI}$ which generalizes the usual semidirect product of groups. Given a variety $\mathcal{V}$ of groups, we denote by $\mathcal{CS}(\mathcal{V})$ the variety of all completely simple semigroups whose subgroups belong to $\mathcal{V}$.

Main theorem. *A regular semigroup S is E-solid and locally inverse if and only if S is embeddable into a λ-semidirect product of a completely simple semigroup by an inverse semigroup. Moreover, if this is the case and the idempotent γ-classes of S belong to $\mathcal{CS}(\mathcal{V})$ for some variety $\mathcal{V}$ of groups then S is embeddable into a λ-semidirect product of a member of $\mathcal{CS}(\mathcal{V})$ by S/γ.*

This is a joint work with I. SZITTYAI.