

Fuzzy automata: State reduction methods

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In this talk we give an overview of our recent research on state reduction of fuzzy automata [2, 3, 7].

State reduction of fuzzy automata has been studied by many authors. All of them have dealt with classical fuzzy automata over the Gödel structure and reduction has been done using crisp equivalence relations. In [2, 3, 7] we have made several innovations. We have studied fuzzy automata over a more general structure of truth values, over a complete residuated lattice, we have shown that better reductions can be attained if we use fuzzy equivalences instead of the crisp ones, and even better if we use fuzzy quasi-orders instead of fuzzy equivalences.

We have established close relationships between state reduction of a fuzzy automaton and resolution of a particular system of fuzzy relation equations, including fuzzy transition relations, fuzzy sets of initial and terminal states, and an unknown fuzzy relation which is required to be a fuzzy equivalence or a fuzzy quasi-order. This system, called the *general system*, has at least one solution, the equality relation, but to attain the best possible reduction of the considered fuzzy automaton we have to find as big a solution as possible. The general system do not necessarily have the greatest solution, and it consists of infinitely many equations, so finding its nontrivial solutions is a very difficult task. For that reason we direct our attention to some instances of the general system, which have to be as general as possible, but which have to be easier to solve. From a practical point of view, these instances have to consist of finitely many equations.

The most interesting instances of the general system are two systems whose solutions in the sets of fuzzy equivalences and fuzzy quasi-orders are called right and left invariant. These fuzzy relations are common generalizations of right and left invariant equivalences and quasi-orders on non-deterministic automata, used by Ilie, Yu, Champarnaud, Coulon and others, in state reduction of non-deterministic automata, as well as

of the so-called congruences on fuzzy automata, used by Petković [6] in state reduction of fuzzy automata.

In [2, 3] we have proved that each fuzzy automaton over a complete residuated lattice possesses the greatest right and left invariant fuzzy equivalences, which provide the best reductions by means of fuzzy equivalences of these types. We have also given effective iteration procedures for computing these fuzzy equivalences, which work if the underlying structure of truth values is a locally finite residuated lattice. Moreover, we have made iteration procedures for computing the greatest right and left invariant crisp equivalences, which work if the underlying structure of truth values is any (not necessarily locally finite) complete residuated lattice, and even if it is any lattice-ordered monoid, but we show that these crisp equivalences give worse reductions than their fuzzy counterparts. Similar results have been obtained in [7] for right and left invariant fuzzy quasi-orders. In the general case, fuzzy quasi-orders and fuzzy equivalences are equally good in state reduction, but we have shown that right and left invariant fuzzy quasi-orders give better reductions than right and left invariant fuzzy equivalences.

In [3, 7] we have also studied special types of right and left invariant fuzzy equivalences and quasi-orders, called strongly right and left invariant ones. We have shown that they give worse reductions than right and left invariant fuzzy equivalences and quasi-orders, but we give algorithms for computing the greatest strongly right and left invariant fuzzy equivalences and quasi-orders which work if the underlying structure of truth values is any (not necessarily locally finite) complete residuated lattice. On the other hand, in [7] we have studied generalizations of right and left invariant fuzzy quasi-orders, called weakly right and left invariant ones. They give better reductions than right and left invariant ones, but they are harder for computing. Namely, weakly right and left invariant fuzzy quasi-orders on a fuzzy recognizer are defined by systems of fuzzy relation equations whose resolution involves determination of this fuzzy recognizer and its reverse fuzzy recognizer by means of the accessible fuzzy subset construction [4, 5]. Similarly to non-deterministic automata, determinization of fuzzy automata may cause exponential growth in the number of states of the resulting deterministic automaton.

We have also shown that even better results in state reduction can be attained alternating reductions by means of the greatest right and left invariant fuzzy quasi-orders and fuzzy equivalences [3, 7]. Namely, a fuzzy automaton reduced by means of the greatest right invariant fuzzy quasi-order can not be further reduced using right invariant fuzzy quasi-orders, but it can be reduced by means of the greatest left invariant ones, and vice versa. This is also true for weakly right and left invariant fuzzy quasi-orders and fuzzy equivalences. It is worth noting that there are examples of non-deterministic automata whose reductions by means of right and left invariant equivalences give a polynomial decrease in the number of states, but alternate reduction causes an exponential decrease.

Let us also note that right invariant equivalences have been studied in many areas of computer science and mathematics, such as modal logic, concurrency theory, set theory, formal verification, model checking, etc., under the name bisimulation equivalence. Many algorithms have been proposed to compute the greatest bisimulation equivalence on a given labelled graph or a labelled transition system (non-deterministic automaton), and the faster algorithms are based on the crucial equivalence between the greatest bisimulation equivalence and the relational coarsest partition problem.

The talk reports a joint work with M. ĆIRIĆ (Niš), J. IGNJATOVIĆ (Niš) and T. PETKOVIĆ (Turku) [2, 3, 7].

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