On the representation of lattices by congruence lattices of semigroups

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Recall that an element a of a complete lattice L is called *compact* if, for any set $X \subseteq L$ with $a \leq \bigvee X$, there exists a finite subset $X' \subseteq X$ such that $a \leq \bigvee X'$. A complete lattice is called *algebraic* if every element in it is the join of some compact elements. The well-known result of G. Grätzer and E. T. Schmidt [1] states that every algebraic lattice is represented as the congruence lattice of an algebra. However, this is not true if we require the finiteness of the similarity type of the corresponding algebra (see [2]). The main theorem of [3] states that if the unit of an algebraic lattice L is compact, then L is represented as the congruence lattice of some groupoid. The following problem is open: is every distributive algebraic lattice isomorphic to the congruence lattice of a groupoid or, more specifically, of a semigroup? We proved the following theorem.

Theorem. *Every distributive algebraic lattice whose compact elements form a lattice with unit is isomorphic to the congruence lattice of a semigroup.*

The talk reports a joint work with V. B. REPNITSKIĬ.

REFERENCES

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