A note on Lallement's lemma

ŽARKO POPOVIĆ
Faculty of Economics, University of NIŠ
zpopovic@eknfak.ni.ac.rs

By $\mathbf{Z}^+$ we denote the set of all positive integers and by $E(S)$ we denote the set of all idempotents of a semigroup S. An element a of a semigroup S is regular, in the sense of J. von Neumann, if there exists $x \in S$ such that $a = axa$. A semigroup S is regular if all of its elements are regular. A semigroup S is π-regular if some power of any element is regular.

A congruence relation ξ on a semigroup S is idempotent-consistent (or idempotent-surjective) if for every idempotent class $a\xi$ of S/ξ there exists $e \in E(S)$ such that $a\,\xi\,e$. This property appears in the conclusion of Lallement's lemma. A semigroup is idempotent-consistent if all of its congruences enjoy this property. These notions were explored by P. M. Higgins (1992, 1998), P. M. Edwards (1983), P. M. Edwards, P. M. Higgins and S. J. L. Kopamu (2001), S. Bogdanović (1984), and H. Mitsch (1996, 1997).

In this talk, for $m, n \in \mathbf{Z}^+$, we define a relation $\tau_{(m,n)}$ by

$$(a, b) \in \tau_{(m,n)} \iff (\forall x \in S^m)(\forall y \in S^n)(\exists k \in \mathbf{Z}^+)\ (xay)^k = (xby)^k,$$

which is a congruence relation on any semigroup S. Using the congruence relation $\tau_{(m,n)}$ we will give a new version of Lallement's lemma. So, we prove that every semigroup S for which $S/\tau_{(m,n)}$ is π-regular is idempotent-consistent. This result is a generalization of the results obtained by P. M. Edwards, P. M. Higgins and S. J. L. Kopamu in 2001.

The talk reports a joint work with S. BOGDANOVIĆ (University of Niš).