Structure in mappings on the random graph

MICHAEL PINSKER
LMNO Laboratory, University of CAEN
marula@gmx.at

We consider functions acting on the random graph. More specifically, we are interested in (topologically) closed transformation monoids and closed permutation groups which contain $\mathrm{Aut}(G)$, the automorphism group of the random graph. With the help of the Nešetřil-Rödl theorem which states that the set of finite ordered structures of a given signature forms a Ramsey class, one can find well-behaved parts in any mapping on the random graph. In particular, we proved:

- Every closed transformation monoid which contains $\mathrm{Aut}(G)$ either contains one of three very primitive functions which I will define in my talk, or is a "disguised group" in the sense that it is generated by the largest group which it contains. For the latter case there are precisely five possibilities, i.e., there exist five closed permutation groups which contain $\mathrm{Aut}(G)$. (The group case was already resolved by Simon Thomas using a different approach.)
- Monoids (groups) containing $\mathrm{Aut}(G)$ correspond to reducts of the random graph, i.e., to structures with a first-order definition in the random graph, in a natural way. By means of this correspondence, properties of these monoids (groups) translate into properties of reducts. Our theorem implies some model-theoretic corollaries which exploit this correspondence. For example we obtain that all reducts of the random graph are model-complete.

The talk reports a joint work with M. BODIRSKY.