Some new tractable cases of the CSP

PETAR MARKOVIĆ

Department of Mathematics and Informatics
Faculty of Science, University of NOVI SAD
`pera@dmi.uns.ac.rs, pera@im.ns.ac.yu`

In this lecture I will review some recent advances on the Constraint Satisfaction Problem Dichotomy conjecture [3]. The *fixed-template Constraint Satisfaction Problem* CSP(**A**) is the decision problem parametrized by a fixed finite relational structure **A** of a finite similarity type (called the template). The problem has an input consisting of a finite relational structure **B** similar to **A** and asks if there exists a homomorphism from **B** to **A**. The Dichotomy Conjecture states that for each **A**, the time complexity of CSP(**A**) is either tractable, or NP-complete.

The work I am presenting is inspired by an analysis of small examples of **A**, which led to a discovery of a few new algorithms extending the known tractable cases. These algorithms are all hybrids between the semilattice subcase of bounded width and the linear algebraic (Mal'cev) algorithm. This lecture will focus on the algorithms we obtained, rather than the algebras and structures on which they can be applied.

To motivate this, note that the idempotent clones on a two-element set fall within the union of Mal'cev, near-unanimity and semilattice cases, and that L. Barto and M. Kozik have fully explained in [1] the case when the Mal'cev situation doesn't occur, while a significant section (according to a 2008 conjecture by M. Valeriote, everything) in the case when the semilattice case doesn't occur is covered by results of [2] and [4]. The results presented are work in progress exploring the case when the near-unanimity situation does not occur.

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