Property ($\hbar$) and cellularity of complete Boolean algebras

MILOŠ KURILIĆ
Deaprtment of Mathematics and Informatics
Faculty of Science, University of NOVI SAD
milos@dmi.uns.ac.rs

A complete Boolean algebra $\mathbb{B}$ satisfies property $(\hbar)$ if and only if each sequence x in $\mathbb{B}$ has a subsequence y such that the equality $\limsup z_n = \limsup y_n$ holds for each subsequence z of y. This property, providing an explicit definition of the a posteriori convergence in complete Boolean algebras with the sequential topology and a characterization of sequential compactness of such spaces, is closely related to the cellularity of Boolean algebras. Here we determine the position of property $(\hbar)$ with respect to the hierarchy of conditions of the form κ-cc. So, answering a question from [M. S. Kurilić, A. Pavlović, A posteriori convergence in complete Boolean algebras with the sequential topology, *Ann. Pure Appl. Logic* **148** (2007), 49-62] we show that "$\mathfrak{h}$-cc $\Rightarrow$ $(\hbar)$" is not a theorem of ZFC and that there is no cardinal $\mathfrak{k}$, definable in ZFC, such that "$\mathfrak{k}$-cc $\Leftrightarrow$ $(\hbar)$" is a theorem of ZFC. Also, we show that the set

$$\{\kappa : \text{ each } \kappa\text{-cc c.B.a. has } (\hbar)\}$$

is equal either to $[0, \mathfrak{h})$ or to $[0, \mathfrak{h}]$ and that both values are consistent, which, with the known equality

$$\{\kappa : \text{ each c. B. a. having } (\hbar) \text{ has the } \kappa\text{-cc}\} = [\mathfrak{s}, \infty)$$

completes the picture.

The talk reports a joint work with S. TODORČEVIĆ (University Paris VII & University of Toronto).