Fuzzy network analysis: Regular fuzzy equivalences and bisimulation

IVANA JANČIĆ
Department of Mathematics and Informatics
Faculty of Science, University of NIŠ
ivanajancic84@gmail.com

Network analysis has originated as a branch of sociology and mathematics which provides formal models and methods for the systematic study of social structures, and it has an especially long tradition in sociology, social psychology and anthropology. But, concepts of network analysis capture the common properties of all networks and its methods are applicable to the analysis of networks in general. For that reason, methods of network analysis are nowadays increasingly applied to many networks which are not social networks but share a number of commonalities with social networks, such as the hyperlink structure on the Web, the electric grid, computer networks, information networks or various large-scale networks appearing in nature.

The key difference between network analysis and other approaches is the focus on relationships between actors rather than the attributes of individual actors. Network analysis takes a global view on network structures, based on the belief that types and patterns of relationships emerge from individual connectivity and that the presence (or absence) of such types and patterns have substantial effects on the network and its constituents. In particular, the network structure provides opportunities and imposes constraints on the individual actors by determining the transfer or flow of resources (material or immaterial) across the network. Such an approach requires a set of methods and analytic concepts that are distinct from the methods of traditional statistics and data analysis. The natural means to model networks mathematically is provided by the notions of graphs, relations and matrices, and methods of network analysis primarily originate from graph theory, semigroup theory and linear algebra. This formality served network analysis to reduce the vagueness in formulating and testing its theories, and contributed to more coherence in the field by allowing researchers to carry out more precise discussions in the literature and to compare results across studies.

However, vagueness in social and many other networks can not be completely avoided, since relations between nodes are in essence vague. This vagueness can be overcame applying fuzzy approach to network analysis, but still, only few authors dealt with this topic (cf. [3, 4, 5, 9, 10]).

We define a fuzzy network as a fuzzy structure $\mathscr{A} = (A, \{Q_i\}_{i\in I})$, where A is a non-empty set of nodes (usually finite), and $\{Q_i\}_{i\in I}$ is a family of fuzzy relations on A. Alternatively, fuzzy networks can be treated as directed fuzzy multigraphs, directed labelled fuzzy graphs (with labels taken from the index set I), or as fuzzy automata (with I as its input alphabet). In the positional analysis of networks, whose aim is to find similarities between nodes in a network, one of the the most studied notions is a regular equivalence, where two nodes are considered to be regularly equivalent if they are equally related to equivalent others. This notion has been extended to the fuzzy framework by Fan, Liau and Lin [4, 5], who have defined a regular fuzzy equivalence on a fuzzy network $\mathscr{A}$ as any fuzzy equivalence on A which is a solution to the system of fuzzy relation equations $R \circ Q_i = Q_i \circ R$, $i \in I$ (R is an unknown fuzzy relation). In [7] regular fuzzy equivalences have been studied in a more general setting, where complete residuated lattices have been used as the underlying structures of truth values, and the above system has been split into two systems of fuzzy relation inequalities: $R \circ Q_i \leq Q_i \circ R$, $i \in I$, and $Q_i \circ R \leq R \circ Q_i$, $i \in I$. It has been proved that these two systems have the greatest solutions in the set of all fuzzy relations on A, and that these greatest solutions are fuzzy quasi-orders. It has been also shown that on the set of all fuzzy quasi-orders on A these systems of fuzzy relation inequalities are respectively equivalent to the following systems of fuzzy relation equations: $R \circ Q_i \circ R = Q_i \circ R$, $i \in I$, and $R \circ Q_i \circ R = R \circ Q_i$, $i \in I$. These two systems of fuzzy relation equations have shown themselves to be exceptionally convenient for construction of the greatest solutions to the mentioned systems of fuzzy relation equations and inequalities in the sets of all fuzzy quasi-orders and all fuzzy equivalences on A, and iterative procedures for computing these greatest solutions have been given in [7]. They finish in a finite number of steps if the underlying structure $\mathscr{L}$ of truth values is locally finite, but they do not necessary finish in a finite number of steps if $\mathscr{L}$ is not locally finite. In particular, the procedures work for classical fuzzy automata over the Gödel structure, but they do not necessary work for fuzzy automata over the Goguen (product) structure.

It is worth noting that if a fuzzy network $\mathscr{A}$ is treated as a fuzzy automaton, then the greatest fuzzy equivalences which are solutions to $R \circ Q_i \leq Q_i \circ R$, $i \in I$, and $Q_i \circ R \leq R \circ Q_i$, $i \in I$, are just forward and backward bisimulation equivalences studied in [6], or right and left invariant fuzzy equivalences studied in [1, 2]. As we have already mentioned, regular fuzzy equivalences are used to capture similarity between nodes in a fuzzy network. To capture similarity between nodes of two different networks, in [8] we have introduced and studied the concept of a regular bisimulation. The obtained results will be also presented in this talk.

The talk reports a joint work with J. IGNJATOVIĆ (Niš) and M. ĆIRIĆ (Niš) [7, 8].

REFERENCES

[1] M. Ćirić, A. Stamenković, J. Ignjatović, T. Petković, Factorization of fuzzy automata, in: E. Csuhaj-Varju, and Z. Ésik (eds.), FCT 2007, Lecture Notes in Computer Science 4639 (2007) 213–225.

[2] M. Ćirić, A. Stamenković, J. Ignjatović, T. Petković, Fuzzy relation equations and reduction of fuzzy automata, Journal of Computer and System Sciences (submitted for publication).

[3] G. B. Davis, K. M. Carley, Clearing the FOG: Fuzzy, overlapping groups for social networks, Social Networks 30 (2008) 201–212.

[4] T. F. Fan, C. J. Liau, T. Y. Lin, Positional analysis in fuzzy social networks, in: Proceedings of the Third IEEE International Conference on Granular Computing, 2007, pp. 423–428.

[5] T. F. Fan, C. J. Liau, T. S. Lin, A theoretical investigation of regular equivalences for fuzzy graphs, International Journal of Approximate Reasoning 49 (2008), 678–688.

[6] M. Ćirić, J. Ignjatović, N. Damljanović, M. Bašić, Bisimulations for fuzzy automata, Fuzzy Sets and Systems (submitted for publication).

[7] J. Ignjatović, M. Ćirić, S. Bogdanović, Fuzzy relation equations and positional analysis of fuzzy networks, (in preparation).

[8] I. Jančić, J. Ignjatović, M. Ćirić, Positional analysis of fuzzy networks and regular bisimulations, (in preparation).

[9] P.S. Nair, S. Sarasamma, Data mining through fuzzy social network analysis, in: Proceedings of the 26th Annual Meeting of the North American Fuzzy Information Processing Society, 2007, pp. 251–255.

[10] M. E. J. Newman, Analysis of weighted networks, Physical Review E 70 (2004), 056131:1–9.