Islands

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The aim of the talk is to present some new investigations of islands, which are particular combinatorial objects. These investigations are all connected with lattices.

Given a square grid in a big rectangle, where each cell is filled with a real number, its height. A rectangle on a grid is called a *rectangular island* if and only if there is a possible water level such that the rectangle is an island in the usual sense [2]. In other words, rectangular islands are rectangles in the rectangular board whose entries are greater than the values of all neighboring cells. The notion comes from information theory [4].

The talk starts with a summary about former results on islands. Then a surprising exact formula and its proof will be presented for the maximum number of hypercubic islands (i.e. sub-Boolean algebras that are intervals) in a big hypercube (i.e. in a Boolean algebra $\{0,1\}^n$). In this problem, the set of cells - the board - consists of all vertices of a hypercube, in other words the elements of a Boolean algebra $\{0,1\}^n$. We consider two cells neighboring if their Hamming distance is 1. In the last part of the talk, we consider a board to be a lattice valued relation and we investigate it by using its particular (ordinary) sub-relations whose characteristic functions are called cut-relations. Special boards are *rectangular lattice valued relations,* whose cut-relations consist of distant (disjoint) rectangles. It turned out that having in mind islands, one can consider only rectangular lattice valued relations. Characterization theorem for rectangular lattice valued relations will be given. We report also about the maximum number of different cuts for two cases: the co-domain of the lattice-valued relation is the set of natural numbers or the interval $[0,1]$ of real numbers.

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