Šárka Stejskalová

Department of Logic, Charles University / Institute of Mathematics,
Czech Academy of Sciences; Prague
sarka.stejskalova@ff.cuni.cz

Some compactness principles and their indestructibility

Compactness principles at small cardinals provide certain degree of large cardinal strength without requiring inaccessibility of the given cardinals. For instance, the weak compactness of κ implies its inaccessibility and also the tree property. Unlike inaccessibility, the tree property can still hold at small cardinals and can be seen as the "combinatorial core" of weak compactness. Out of general interest, and also as a technical tool in forcing constructions, it is useful to study how easy or difficult it is to destroy these principles by forcing.

In the talk we will survey recent results regarding the preservation (or indestructibility) of some compactness principles by forcing notions. We will focus primarily on the negation of the weak Kurepa hypothesis at ω_1, the tree property at ω_2, stationary reflection (in different versions) at ω_2 and the guessing model principle at ω_2, and their preservation by ccc and σ-centered forcings. We will also discuss relationships between these principles, and their efect on the continuum function.

In the first part of the talk we will focus on the preservation of these principles over the Mitchell model – which is the standard model for obtaining these principles. In the second part of the talk we will extend our scope to study preservation not only for specific models (such as Mitchell's), but over theories, the main example being the guessing model principle, and stationary reflection.