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Geometric Set Theory

Geometric set theory studies transitive models of set theory with respect to their extensional agreement. Part of the subject deals with Borel equivalence relations on Polish spaces. Given an equivalence E on a space X and a configuration $\{M_i : i \in I\}$ of transitive models of set theory, one asks whether it is possible for an E-equivalence class to have representatives in some models of the configuration and fail to be represented in others. The answer to this question greatly varies with the nature of the equivalence relation and the configuration in question, and the resulting differences can be used to prove non-reducibility theorems for various Borel equivalence relations.

Another part deals with independence results in choiceless Zermelo–Fraenkel (ZF) set theory. We study ZF independence results between Σ^2_1 consequences of the Axiom of Choice which are connected to various contemporary concerns of descriptive set theory and analysis. A detailed structure appears, with some fracture lines running parallel to existing combinatorial, algebraic, or analytic concepts. Given a Σ^2_1 sentence $\phi = \exists A \subset X\ \psi(A)$, a consequence of the Axiom of Choice in which X is a Polish space and ψ is a formula quantifying only over elements of Polish spaces, and given a configuration $\{M_i : i \in I\}$ of transitive models of set theory with choice, is there a set $A \subset X$ such that in all (or many) models M in the configuration, $A \cap M \in M$ and $A \cap M$ is a witness to ϕ in the model M? The answer to this question is surprisingly varied and successful in separating consequences of the Axiom of Choice of this syntactical complexity.

The models of ZF we use for our independence results are extensions of the symmetric Solovay model by simply definable σ-closed forcings. As a result, they are all models of DC, the Axiom of Dependent Choice. Given Σ^2_1 sentences ϕ_0, ϕ_1, there is usually a canonical choice for a forcing which should generate a model of ZF+DC+$\neg\phi_0$+ϕ_1, if this theory is consistent. The whole analysis of the forcing in question takes place in ZFC, using central concepts of the fields related to the Σ_1^2 sentences in question.