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Congruence of ultrafilters

As usual, let $\beta\mathbb{N}$ denote the set of ultrafilters on the set \mathbb{N} of natural numbers, with elements of \mathbb{N} identified with principal ultrafilters. A quasiorder $\tilde{|}$ on $\beta\mathbb{N}$, a natural extension of the divisibility relation $|$ on \mathbb{N} , was considered in several papers ([3],[4]). It turned out that nonstandard methods can lead to better understanding of this relation ([5],[6]).

One can also extend to $\beta\mathbb{N}$ the congruence relation modulo an integer, and we examine to what extent this relation agrees with $\tilde{|}$. Afterwards we propose a way to define congruence modulo an ultrafilter and find its nonstandard characterization. Using iterated nonstandard extensions of \mathbb{N} , and in particular the notion of tensor pairs (from [1] and [2]), we also introduce so-called strong congruence. This relation is perhaps less natural, but has nicer properties with respect to $\tilde{|}$.

- [1] Di Nasso, M., Hypernatural numbers as ultrafilters. in: Nonstandard Analysis for the Working Mathematician, Springer, Dordrecht, 2015, 443–474.
- [2] Luperi Baglini, L., Hyperintegers and nonstandard techniques in combinatorics of numbers. PhD thesis, University of Siena (2012).
- [3] Šobot, B., Divisibility in the Stone-Čech compactification. Rep. Math. Logic Vol. 50 (2015), 53–66.
- [4] Šobot, B., $\tilde{|}$ -divisibility of ultrafilters. Ann. Pure Appl. Logic Vol. 172 (2021), No.1.

- [5] Šobot, B., Divisibility in $\beta\mathbb{N}$ and ${}^*\mathbb{N}$. Rep. Math. Logic Vol. 54 (2019), 65–82.
- [6] Šobot, B., More about divisibility in $\beta\mathbb{N}$. MLQ Math. Log. Q. Vol. 67 (2021), No.1, 77–87.
- [7] Šobot, B., Congruence of ultrafilters.. J. Symbolic Logic Vol. 86 (2021), No.2, 746–761.