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A model in which wellorderings of the reals first appear at a given projective level

Joint work with Vassily Lyubetsky

Theorem. *Let $n \geq 3$. There is a generic extension $L[G]$ of L in which it is true that there is a good lightface $\varDelta^1_n$ wellordering of the reals, but there is no any boldface $\mathbf{\Sigma}^1_{n-1}$ wellordering of the reals.*

We make use of the model $L[G]$ defined in [1] in which the Separation principle holds for lightface $\varSigma^1_n$ sets of integers (for a given $n \geq 3$). The model extends L by a generic transfinite sequence of reals x_α, $\alpha < \omega_1^L$. Each x_α is P_α-generic over L, where $P_\alpha \in L$ is a forcing that consists of perfect trees and is rather similar to Jensen's forcing as in [2, 28A]. The sequence of reals x_α turns out to be $\varDelta^1_n$ in the extension, which allows to define a good $\varDelta^1_n$ wellordering of the reals. On the other hand, a special, *generic* in some sense construction of P_α in L allows to obscure the mutual differences between these forcing notions in such a way that no $\mathbf{\Sigma}^1_{n-1}$ wellordering even of the set of all reals x_α exists in the extension.

A weaker result has just appeared in [2]. It has the negative part in the form of nonexistence of good $\varDelta^1_{n-1}$ wellorderings of the reals.

[1] Kanovei, V., Lyubetsky, A., Models of set theory in which separation theorem fails. Izvestiya: Mathematics, Vol. 85 No. 6 (2021), 1181–1219.

[2] Jech, T., Set theory, Springer, 2003.

[3] Kanovei, V., Lyubetsky, A., A model in which wellorderings of the reals first appear at a given projective level. Axioms, Vol. nn No. n (2022), 1–13.