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On the resolvability of product spaces

Joint work with L. Soukup and Z. Szentmiklóssy

All spaces below are T_0 and crowded (i.e. have no isolated points).

For $n \leq \omega$ let $M(n)$ be the statement that there are n measurable cardinals and $\Pi(n)$ ($\Pi^+(n)$) that there are $n+1$ (0-dimensional T_2) spaces whose product is irresolvable. We prove that

1. $M(1)$, $\Pi(1)$ and $\Pi^+(1)$ are equiconsistent;
2. if $1 < n < \omega$ then $CON(M(n))$ implies $CON(\Pi^+(n))$;
3. $CON(M(\omega))$ implies the consistency of having infinitely many 0-dimensional T_2-spaces such that the product of any finitely many of them is irresolvable.

These results settle old problems of Malychin.

Concerning an even older question of Ceder and Pearson, we show that the following are consistent modulo a measurable cardinal:

1. There is a 0-dimensional T_2 space X with $\omega_2 \leq |X| = \Delta(X) \leq 2^{\omega_1}$ whose product with any countable space is not ω_2-resolvable.
2. There is a monotonically normal space X with $|X| = \Delta(X) = \aleph_\omega$ whose product with any countable space is not ω_1-resolvable.

These significantly improve a result of Eckertson.