## On $\sigma$ -countably tight spaces

#### István Juhász

Alfréd Rényi Institute of Mathematics, Hungarian Academy of Sciences

#### SETTOP 2016

June 23, 2016

## 6TH EUROPEAN SET THEORY CONFERENCE

### JULY 3-7, 2017

### MTA RÉNYI INSTITUTE OF MATHEMATICS, BUDAPEST

www.renyi.hu

István Juhász (MTA Rényi Institute)

A B F A B F

A .

æ

イロト イヨト イヨト イヨト

2

イロト イヨト イヨト イヨト

DEFINITION. A space is  $\sigma$ -countably tight (=  $\sigma$ -CT), resp. *fin*-CT, if it is covered by countably many, resp. finitely many CT subspaces.

DEFINITION. A space is  $\sigma$ -countably tight (=  $\sigma$ -CT), resp. *fin*-CT, if it is covered by countably many, resp. finitely many CT subspaces.

## THEOREM (R. de la Vega)

Every CT homogeneous compactum has cardinality c.

< ロ > < 同 > < 回 > < 回 >

DEFINITION. A space is  $\sigma$ -countably tight (=  $\sigma$ -CT), resp. *fin*-CT, if it is covered by countably many, resp. finitely many CT subspaces.

## THEOREM (R. de la Vega)

Every CT homogeneous compactum has cardinality c.

compactum = infinite compact  $T_2$  space

< ロ > < 同 > < 回 > < 回 >

DEFINITION. A space is  $\sigma$ -countably tight (=  $\sigma$ -CT), resp. *fin*-CT, if it is covered by countably many, resp. finitely many CT subspaces.

## THEOREM (R. de la Vega)

Every CT homogeneous compactum has cardinality c.

compactum = infinite compact  $T_2$  space

#### PROBLEM

Are  $\sigma$ -CT (or *fin*-CT) homogeneous compacta of cardinality c?

・ロト ・ 四ト ・ ヨト ・ ヨト

DEFINITION. A space is  $\sigma$ -countably tight (=  $\sigma$ -CT), resp. *fin*-CT, if it is covered by countably many, resp. finitely many CT subspaces.

## THEOREM (R. de la Vega)

Every CT homogeneous compactum has cardinality c.

compactum = infinite compact  $T_2$  space

#### PROBLEM

Are  $\sigma$ -CT (or fin-CT) homogeneous compacta of cardinality c?

EXAMPLES. 1) Set  $\sigma_i = \{ x \in \{0, 1\}^{\kappa} : |\{\alpha < \kappa : x(\alpha) \neq i\}| < \omega \}.$ 

DEFINITION. A space is  $\sigma$ -countably tight (=  $\sigma$ -CT), resp. *fin*-CT, if it is covered by countably many, resp. finitely many CT subspaces.

## THEOREM (R. de la Vega)

Every CT homogeneous compactum has cardinality c.

compactum = infinite compact  $T_2$  space

#### PROBLEM

Are  $\sigma$ -CT (or *fin*-CT) homogeneous compacta of cardinality c?

EXAMPLES. 1) Set  $\sigma_i = \{x \in \{0, 1\}^{\kappa} : |\{\alpha < \kappa : x(\alpha) \neq i\}| < \omega\}$ . Then  $\sigma_0 \cup \sigma_1$  is a  $\sigma$ -compact subgroup of  $\mathbb{C}_{\kappa} = \{0, 1\}^{\kappa}$ , both  $\sigma_0$  and  $\sigma_1$  are Frèchet, hence CT, but  $t(\sigma_0 \cup \sigma_1) = |\sigma_0 \cup \sigma_1| = \kappa$ .

DEFINITION. A space is  $\sigma$ -countably tight (=  $\sigma$ -CT), resp. *fin*-CT, if it is covered by countably many, resp. finitely many CT subspaces.

## THEOREM (R. de la Vega)

Every CT homogeneous compactum has cardinality c.

compactum = infinite compact  $T_2$  space

#### PROBLEM

Are  $\sigma$ -CT (or *fin*-CT) homogeneous compacta of cardinality c?

EXAMPLES. 1) Set  $\sigma_i = \{x \in \{0, 1\}^{\kappa} : |\{\alpha < \kappa : x(\alpha) \neq i\}| < \omega\}$ . Then  $\sigma_0 \cup \sigma_1$  is a  $\sigma$ -compact subgroup of  $\mathbb{C}_{\kappa} = \{0, 1\}^{\kappa}$ , both  $\sigma_0$  and  $\sigma_1$  are Frèchet, hence CT, but  $t(\sigma_0 \cup \sigma_1) = |\sigma_0 \cup \sigma_1| = \kappa$ .

2) The compactum  $\omega_1 + 1$  is 2-CT with  $t(X) > \omega$ .

DEFINITION. A space is  $\sigma$ -countably tight (=  $\sigma$ -CT), resp. *fin*-CT, if it is covered by countably many, resp. finitely many CT subspaces.

## THEOREM (R. de la Vega)

Every CT homogeneous compactum has cardinality c.

compactum = infinite compact  $T_2$  space

#### PROBLEM

Are  $\sigma$ -CT (or *fin*-CT) homogeneous compacta of cardinality c?

EXAMPLES. 1) Set  $\sigma_i = \{x \in \{0, 1\}^{\kappa} : |\{\alpha < \kappa : x(\alpha) \neq i\}| < \omega\}$ . Then  $\sigma_0 \cup \sigma_1$  is a  $\sigma$ -compact subgroup of  $\mathbb{C}_{\kappa} = \{0, 1\}^{\kappa}$ , both  $\sigma_0$  and  $\sigma_1$  are Frèchet, hence CT, but  $t(\sigma_0 \cup \sigma_1) = |\sigma_0 \cup \sigma_1| = \kappa$ .

2) The compactum  $\omega_1 + 1$  is 2-CT with  $t(X) > \omega$ .

æ

イロト イヨト イヨト イヨト

## THEOREM 1.

Assume that the compactum X is the union of countably many dense CT subspaces,

## THEOREM 1.

Assume that the compactum X is the union of countably many dense CT subspaces, moreover  $X^{\omega}$  is homogeneous.

A > + = + + =

## THEOREM 1.

Assume that the compactum X is the union of countably many dense CT subspaces, moreover  $X^{\omega}$  is homogeneous. Then  $|X| \leq \mathfrak{c}$ .

< ロ > < 同 > < 回 > < 回 >

Assume that the compactum X is the union of countably many dense CT subspaces, moreover  $X^{\omega}$  is homogeneous. Then  $|X| \leq \mathfrak{c}$ .

#### THEOREM 2.

If X is a *fin*-CT homogeneous compactum then |X| = c.

Assume that the compactum X is the union of countably many dense CT subspaces, moreover  $X^{\omega}$  is homogeneous. Then  $|X| \leq \mathfrak{c}$ .

#### THEOREM 2.

If X is a *fin*-CT homogeneous compactum then |X| = c.

On the proofs:

Assume that the compactum X is the union of countably many dense CT subspaces, moreover  $X^{\omega}$  is homogeneous. Then  $|X| \leq c$ .

#### THEOREM 2.

If X is a *fin*-CT homogeneous compactum then |X| = c.

On the proofs: In both cases, it suffices to show

 $w(X) \leq \mathfrak{c}$  and  $\pi\chi(X) = \omega$ 

Assume that the compactum X is the union of countably many dense CT subspaces, moreover  $X^{\omega}$  is homogeneous. Then  $|X| \leq c$ .

#### THEOREM 2.

If X is a *fin*-CT homogeneous compactum then |X| = c.

On the proofs: In both cases, it suffices to show

 $w(X) \leq \mathfrak{c}$  and  $\pi\chi(X) = \omega$ 

by:

Assume that the compactum X is the union of countably many dense CT subspaces, moreover  $X^{\omega}$  is homogeneous. Then  $|X| \leq c$ .

#### THEOREM 2.

If X is a *fin*-CT homogeneous compactum then |X| = c.

On the proofs: In both cases, it suffices to show

 $w(X) \leq \mathfrak{c}$  and  $\pi\chi(X) = \omega$ 

by:

#### THEOREM (van Mill)

If X is a power homogeneous compactum then  $|X| \le w(X)^{\pi\chi(X)}$ .

Assume that the compactum X is the union of countably many dense CT subspaces, moreover  $X^{\omega}$  is homogeneous. Then  $|X| \leq c$ .

#### THEOREM 2.

If X is a *fin*-CT homogeneous compactum then |X| = c.

On the proofs: In both cases, it suffices to show

 $w(X) \leq \mathfrak{c}$  and  $\pi\chi(X) = \omega$ 

by:

#### THEOREM (van Mill)

If X is a power homogeneous compactum then  $|X| \le w(X)^{\pi\chi(X)}$ .

æ

イロト イヨト イヨト イヨト

## THEOREM 3.

(i) No closed subspace of a  $\sigma$ -CT compactum X maps onto  $\mathbb{C}_{\omega_1}$ ,

< ロ > < 同 > < 回 > < 回 >

### THEOREM 3.

(i) No closed subspace of a  $\sigma$ -CT compactum X maps onto  $\mathbb{C}_{\omega_1}$ , hence there is  $x \in X$  with  $\pi \chi(x, X) \leq \omega$ .

< ロ > < 同 > < 回 > < 回 >

### THEOREM 3.

(i) No closed subspace of a  $\sigma$ -CT compactum X maps onto  $\mathbb{C}_{\omega_1}$ , hence there is  $x \in X$  with  $\pi\chi(x, X) \leq \omega$ . So,  $\pi\chi(X) = \omega$  if X is homogeneous.

### THEOREM 3.

(i) No closed subspace of a  $\sigma$ -CT compactum X maps onto  $\mathbb{C}_{\omega_1}$ , hence there is  $x \in X$  with  $\pi\chi(x, X) \leq \omega$ . So,  $\pi\chi(X) = \omega$  if X is homogeneous.

(ii) Every  $\sigma$ -CT compactum X has a non-empty subseparable  $G_{\delta}$  subset H

< ロ > < 同 > < 回 > < 回 >

### THEOREM 3.

(i) No closed subspace of a  $\sigma$ -CT compactum X maps onto  $\mathbb{C}_{\omega_1}$ , hence there is  $x \in X$  with  $\pi\chi(x, X) \leq \omega$ . So,  $\pi\chi(X) = \omega$  if X is homogeneous.

(ii) Every  $\sigma$ -CT compactum X has a non-empty subseparable  $G_{\delta}$  subset H i.e. such that  $H \subset \overline{A}$  for some  $A \in [X]^{\omega}$ .

### THEOREM 3.

(i) No closed subspace of a  $\sigma$ -CT compactum X maps onto  $\mathbb{C}_{\omega_1}$ , hence there is  $x \in X$  with  $\pi\chi(x, X) \leq \omega$ . So,  $\pi\chi(X) = \omega$  if X is homogeneous.

(ii) Every  $\sigma$ -CT compactum X has a non-empty subseparable  $G_{\delta}$  subset H i.e. such that  $H \subset \overline{A}$  for some  $A \in [X]^{\omega}$ . Clearly,  $w(H) \leq \mathfrak{c}$ .

イロト イ団ト イヨト イヨト

### THEOREM 3.

(i) No closed subspace of a  $\sigma$ -CT compactum X maps onto  $\mathbb{C}_{\omega_1}$ , hence there is  $x \in X$  with  $\pi\chi(x, X) \leq \omega$ . So,  $\pi\chi(X) = \omega$  if X is homogeneous.

(ii) Every  $\sigma$ -CT compactum X has a non-empty subseparable  $G_{\delta}$  subset H i.e. such that  $H \subset \overline{A}$  for some  $A \in [X]^{\omega}$ . Clearly,  $w(H) \leq \mathfrak{c}$ .

These facts are well-known for CT compacta

(4) (5) (4) (5)

### THEOREM 3.

(i) No closed subspace of a  $\sigma$ -CT compactum X maps onto  $\mathbb{C}_{\omega_1}$ , hence there is  $x \in X$  with  $\pi\chi(x, X) \leq \omega$ . So,  $\pi\chi(X) = \omega$  if X is homogeneous.

(ii) Every  $\sigma$ -CT compactum X has a non-empty subseparable  $G_{\delta}$  subset H i.e. such that  $H \subset \overline{A}$  for some  $A \in [X]^{\omega}$ . Clearly,  $w(H) \leq \mathfrak{c}$ .

These facts are well-known for CT compacta but they are much harder to prove for  $\sigma$ -CT compacta.

(4) (5) (4) (5)

### THEOREM 3.

(i) No closed subspace of a  $\sigma$ -CT compactum X maps onto  $\mathbb{C}_{\omega_1}$ , hence there is  $x \in X$  with  $\pi\chi(x, X) \leq \omega$ . So,  $\pi\chi(X) = \omega$  if X is homogeneous.

(ii) Every  $\sigma$ -CT compactum X has a non-empty subseparable  $G_{\delta}$  subset H i.e. such that  $H \subset \overline{A}$  for some  $A \in [X]^{\omega}$ . Clearly,  $w(H) \leq \mathfrak{c}$ .

These facts are well-known for CT compacta but they are much harder to prove for  $\sigma$ -CT compacta.

#### THEOREM (Pytkeev)

 $L(X_{\delta}) \leq \mathfrak{c}$  for any CT compactum *X*.

### THEOREM 3.

(i) No closed subspace of a  $\sigma$ -CT compactum X maps onto  $\mathbb{C}_{\omega_1}$ , hence there is  $x \in X$  with  $\pi\chi(x, X) \leq \omega$ . So,  $\pi\chi(X) = \omega$  if X is homogeneous.

(ii) Every  $\sigma$ -CT compactum X has a non-empty subseparable  $G_{\delta}$  subset H i.e. such that  $H \subset \overline{A}$  for some  $A \in [X]^{\omega}$ . Clearly,  $w(H) \leq \mathfrak{c}$ .

These facts are well-known for CT compacta but they are much harder to prove for  $\sigma$ -CT compacta.

#### THEOREM (Pytkeev)

 $L(X_{\delta}) \leq \mathfrak{c}$  for any CT compactum *X*.

The result of de la Vega then easily follows from van Mill's.

3

イロト 不得 トイヨト イヨト

## THEOREM 3.

(i) No closed subspace of a  $\sigma$ -CT compactum X maps onto  $\mathbb{C}_{\omega_1}$ , hence there is  $x \in X$  with  $\pi\chi(x, X) \leq \omega$ . So,  $\pi\chi(X) = \omega$  if X is homogeneous.

(ii) Every  $\sigma$ -CT compactum X has a non-empty subseparable  $G_{\delta}$  subset H i.e. such that  $H \subset \overline{A}$  for some  $A \in [X]^{\omega}$ . Clearly,  $w(H) \leq \mathfrak{c}$ .

These facts are well-known for CT compacta but they are much harder to prove for  $\sigma$ -CT compacta.

#### THEOREM (Pytkeev)

 $L(X_{\delta}) \leq \mathfrak{c}$  for any CT compactum *X*.

The result of de la Vega then easily follows from van Mill's.

We do not know if Pytkeev's thm holds for  $\sigma$ - or fin-CT spaces :-(

- 34

## THEOREM 3.

(i) No closed subspace of a  $\sigma$ -CT compactum X maps onto  $\mathbb{C}_{\omega_1}$ , hence there is  $x \in X$  with  $\pi\chi(x, X) \leq \omega$ . So,  $\pi\chi(X) = \omega$  if X is homogeneous.

(ii) Every  $\sigma$ -CT compactum X has a non-empty subseparable  $G_{\delta}$  subset H i.e. such that  $H \subset \overline{A}$  for some  $A \in [X]^{\omega}$ . Clearly,  $w(H) \leq \mathfrak{c}$ .

These facts are well-known for CT compacta but they are much harder to prove for  $\sigma$ -CT compacta.

#### THEOREM (Pytkeev)

 $L(X_{\delta}) \leq \mathfrak{c}$  for any CT compactum *X*.

The result of de la Vega then easily follows from van Mill's.

We do not know if Pytkeev's thm holds for  $\sigma$ - or fin-CT spaces :-(

- 34

æ

イロト イヨト イヨト イヨト

#### THEOREM 4.

István Juhász (MTA Rényi Institute)

æ

イロト イヨト イヨト イヨト

### THEOREM 4.

Let X be a compactum with two covers  $\mathcal{Y}$  and  $\mathcal{H}$  such that

< ロ > < 同 > < 回 > < 回 >

### THEOREM 4.

Let X be a compactum with two covers  $\mathcal{Y}$  and  $\mathcal{H}$  such that

(1)  $|\mathcal{Y}| \leq \mathfrak{c}$ , moreover every  $Y \in \mathcal{Y}$  is CT and dense in X;

### THEOREM 4.

Let X be a compactum with two covers  $\mathcal{Y}$  and  $\mathcal{H}$  such that

- (1)  $|\mathcal{Y}| \leq \mathfrak{c}$ , moreover every  $Y \in \mathcal{Y}$  is CT and dense in X;
- (2) every  $H \in \mathcal{H}$  is a  $G_{\delta}$  of weight  $w(H) \leq \mathfrak{c}$ .

### THEOREM 4.

Let X be a compactum with two covers  $\mathcal{Y}$  and  $\mathcal{H}$  such that

```
(1) |\mathcal{Y}| \leq \mathfrak{c}, moreover every Y \in \mathcal{Y} is CT and dense in X;
```

```
(2) every H \in \mathcal{H} is a G_{\delta} of weight w(H) \leq \mathfrak{c}.
```

Then  $w(X) \leq \mathfrak{c}$ .

< ロ > < 同 > < 回 > < 回 >

### THEOREM 4.

Let X be a compactum with two covers  $\mathcal{Y}$  and  $\mathcal{H}$  such that

```
(1) |\mathcal{Y}| \leq \mathfrak{c}, moreover every Y \in \mathcal{Y} is CT and dense in X;
```

```
(2) every H \in \mathcal{H} is a G_{\delta} of weight w(H) \leq \mathfrak{c}.
```

Then  $w(X) \leq \mathfrak{c}$ .

Pf of THM 1. :

#### THEOREM 4.

Let X be a compactum with two covers  $\mathcal{Y}$  and  $\mathcal{H}$  such that

```
(1) |\mathcal{Y}| \leq \mathfrak{c}, moreover every Y \in \mathcal{Y} is CT and dense in X;
```

```
(2) every H \in \mathcal{H} is a G_{\delta} of weight w(H) \leq \mathfrak{c}.
```

Then  $w(X) \leq \mathfrak{c}$ .

Pf of THM 1. :

– There is 
$$x \in X^{\omega}$$
 with  $\pi \chi(x, X) = \omega$ 

### THEOREM 4.

Let X be a compactum with two covers  $\mathcal{Y}$  and  $\mathcal{H}$  such that

- (1)  $|\mathcal{Y}| \leq \mathfrak{c}$ , moreover every  $Y \in \mathcal{Y}$  is CT and dense in X;
- (2) every  $H \in \mathcal{H}$  is a  $G_{\delta}$  of weight  $w(H) \leq \mathfrak{c}$ .

Then  $w(X) \leq \mathfrak{c}$ .

Pf of THM 1. :

– There is 
$$x \in X^{\omega}$$
 with  $\pi \chi(x, X) = \omega \Rightarrow \pi \chi(X^{\omega}) = \omega$ 

### THEOREM 4.

Let X be a compactum with two covers  $\mathcal{Y}$  and  $\mathcal{H}$  such that

- (1)  $|\mathcal{Y}| \leq \mathfrak{c}$ , moreover every  $Y \in \mathcal{Y}$  is CT and dense in *X*;
- (2) every  $H \in \mathcal{H}$  is a  $G_{\delta}$  of weight  $w(H) \leq \mathfrak{c}$ .

Then  $w(X) \leq \mathfrak{c}$ .

Pf of THM 1. :

- There is 
$$x \in X^{\omega}$$
 with  $\pi \chi(x, X) = \omega \Rightarrow \pi \chi(X^{\omega}) = \omega \Rightarrow \pi \chi(X) = \omega;$ 

### THEOREM 4.

Let X be a compactum with two covers  $\mathcal{Y}$  and  $\mathcal{H}$  such that

- (1)  $|\mathcal{Y}| \leq \mathfrak{c}$ , moreover every  $Y \in \mathcal{Y}$  is CT and dense in *X*;
- (2) every  $H \in \mathcal{H}$  is a  $G_{\delta}$  of weight  $w(H) \leq \mathfrak{c}$ .

Then  $w(X) \leq \mathfrak{c}$ .

Pf of THM 1. :

- There is  $x \in X^{\omega}$  with  $\pi \chi(x, X) = \omega \Rightarrow \pi \chi(X^{\omega}) = \omega \Rightarrow \pi \chi(X) = \omega$ ;

-X, hence  $X^{\omega}$  has a  $G_{\delta}$  of weight  $\leq \mathfrak{c}$ 

< ロ > < 同 > < 回 > < 回 >

### THEOREM 4.

Let X be a compactum with two covers  $\mathcal{Y}$  and  $\mathcal{H}$  such that

- (1)  $|\mathcal{Y}| \leq \mathfrak{c}$ , moreover every  $Y \in \mathcal{Y}$  is CT and dense in *X*;
- (2) every  $H \in \mathcal{H}$  is a  $G_{\delta}$  of weight  $w(H) \leq \mathfrak{c}$ .

Then  $w(X) \leq \mathfrak{c}$ .

Pf of THM 1. :

- There is  $x \in X^{\omega}$  with  $\pi \chi(x, X) = \omega \Rightarrow \pi \chi(X^{\omega}) = \omega \Rightarrow \pi \chi(X) = \omega;$ 

-X, hence  $X^{\omega}$  has a  $G_{\delta}$  of weight  $\leq \mathfrak{c} \Rightarrow X^{\omega}$ , hence X can be covered by them;

3

### THEOREM 4.

Let X be a compactum with two covers  $\mathcal{Y}$  and  $\mathcal{H}$  such that

- (1)  $|\mathcal{Y}| \leq \mathfrak{c}$ , moreover every  $Y \in \mathcal{Y}$  is CT and dense in *X*;
- (2) every  $H \in \mathcal{H}$  is a  $G_{\delta}$  of weight  $w(H) \leq \mathfrak{c}$ .

Then  $w(X) \leq \mathfrak{c}$ .

Pf of THM 1. :

- There is  $x \in X^{\omega}$  with  $\pi \chi(x, X) = \omega \Rightarrow \pi \chi(X^{\omega}) = \omega \Rightarrow \pi \chi(X) = \omega;$ 

-X, hence  $X^{\omega}$  has a  $G_{\delta}$  of weight  $\leq \mathfrak{c} \Rightarrow X^{\omega}$ , hence X can be covered by them;

– we have a countable cover  $\mathcal{Y}$  of X by dense CT subsets.

3

### THEOREM 4.

Let X be a compactum with two covers  $\mathcal{Y}$  and  $\mathcal{H}$  such that

- (1)  $|\mathcal{Y}| \leq \mathfrak{c}$ , moreover every  $Y \in \mathcal{Y}$  is CT and dense in *X*;
- (2) every  $H \in \mathcal{H}$  is a  $G_{\delta}$  of weight  $w(H) \leq \mathfrak{c}$ .

Then  $w(X) \leq \mathfrak{c}$ .

Pf of THM 1. :

- There is  $x \in X^{\omega}$  with  $\pi \chi(x, X) = \omega \Rightarrow \pi \chi(X^{\omega}) = \omega \Rightarrow \pi \chi(X) = \omega;$ 

-X, hence  $X^{\omega}$  has a  $G_{\delta}$  of weight  $\leq \mathfrak{c} \Rightarrow X^{\omega}$ , hence X can be covered by them;

- we have a countable cover  $\mathcal{Y}$  of X by dense CT subsets.
- So, by THM 4.,  $w(X) \leq \mathfrak{c}$ .

э.

### THEOREM 4.

Let X be a compactum with two covers  $\mathcal{Y}$  and  $\mathcal{H}$  such that

- (1)  $|\mathcal{Y}| \leq \mathfrak{c}$ , moreover every  $Y \in \mathcal{Y}$  is CT and dense in *X*;
- (2) every  $H \in \mathcal{H}$  is a  $G_{\delta}$  of weight  $w(H) \leq \mathfrak{c}$ .

Then  $w(X) \leq \mathfrak{c}$ .

Pf of THM 1. :

- There is  $x \in X^{\omega}$  with  $\pi \chi(x, X) = \omega \Rightarrow \pi \chi(X^{\omega}) = \omega \Rightarrow \pi \chi(X) = \omega;$ 

-X, hence  $X^{\omega}$  has a  $G_{\delta}$  of weight  $\leq \mathfrak{c} \Rightarrow X^{\omega}$ , hence X can be covered by them;

- we have a countable cover  $\mathcal{Y}$  of X by dense CT subsets.
- So, by THM 4.,  $w(X) \leq c$ . Now apply van Mill.

### THEOREM 4.

Let X be a compactum with two covers  $\mathcal{Y}$  and  $\mathcal{H}$  such that

- (1)  $|\mathcal{Y}| \leq \mathfrak{c}$ , moreover every  $Y \in \mathcal{Y}$  is CT and dense in *X*;
- (2) every  $H \in \mathcal{H}$  is a  $G_{\delta}$  of weight  $w(H) \leq \mathfrak{c}$ .

Then  $w(X) \leq \mathfrak{c}$ .

Pf of THM 1. :

- There is  $x \in X^{\omega}$  with  $\pi \chi(x, X) = \omega \Rightarrow \pi \chi(X^{\omega}) = \omega \Rightarrow \pi \chi(X) = \omega;$ 

-X, hence  $X^{\omega}$  has a  $G_{\delta}$  of weight  $\leq \mathfrak{c} \Rightarrow X^{\omega}$ , hence X can be covered by them;

- we have a countable cover  $\mathcal{Y}$  of X by dense CT subsets.
- So, by THM 4.,  $w(X) \leq c$ . Now apply van Mill.

æ

イロト イヨト イヨト イヨト

æ

イロト イヨト イヨト イヨト

 $-\pi\chi(X) = \omega$  as X is homogeneous;

э

イロト イ団ト イヨト イヨト

 $-\pi\chi(X) = \omega$  as X is homogeneous;

-X has a cover  $\mathcal{H}$  by  $G_{\delta}$  sets of weight  $\leq \mathfrak{c}$ ;

 $-\pi\chi(X) = \omega$  as X is homogeneous;

-X has a cover  $\mathcal{H}$  by  $G_{\delta}$  sets of weight  $\leq \mathfrak{c}$ ;

 $-X = \cup \mathcal{Y}$  where  $|\mathcal{Y}| < \omega$  and every  $Y \in \mathcal{Y}$  is CT;

3

 $-\pi\chi(X) = \omega$  as X is homogeneous;

- *X* has a cover  $\mathcal{H}$  by  $G_{\delta}$  sets of weight  $\leq \mathfrak{c}$ ;

 $- X = \cup \mathcal{Y}$  where  $|\mathcal{Y}| < \omega$  and every  $Y \in \mathcal{Y}$  is CT;

- there are  $\overline{U}$  regular closed and  $\mathcal{Z} \subset \mathcal{Y}$  such that  $\overline{U} \subset \cup \mathcal{Z}$  and  $Z \cap \overline{U}$  is dense in  $\overline{U}$  for every  $Z \in \mathcal{Z}$ ;

. . . . . . .

 $-\pi\chi(X) = \omega$  as X is homogeneous;

- *X* has a cover  $\mathcal{H}$  by  $G_{\delta}$  sets of weight  $\leq \mathfrak{c}$ ;

 $- X = \cup \mathcal{Y}$  where  $|\mathcal{Y}| < \omega$  and every  $Y \in \mathcal{Y}$  is CT;

- there are  $\overline{U}$  regular closed and  $\mathcal{Z} \subset \mathcal{Y}$  such that  $\overline{U} \subset \cup \mathcal{Z}$  and  $Z \cap \overline{U}$  is dense in  $\overline{U}$  for every  $Z \in \mathcal{Z}$ ;

- hence  $w(\overline{U}) \leq \mathfrak{c}$  by THM 4. ;

. . . . . . .

 $-\pi\chi(X) = \omega$  as X is homogeneous;

- *X* has a cover  $\mathcal{H}$  by  $G_{\delta}$  sets of weight  $\leq \mathfrak{c}$ ;

 $- X = \cup \mathcal{Y}$  where  $|\mathcal{Y}| < \omega$  and every  $Y \in \mathcal{Y}$  is CT;

- there are  $\overline{U}$  regular closed and  $\mathcal{Z} \subset \mathcal{Y}$  such that  $\overline{U} \subset \cup \mathcal{Z}$  and  $Z \cap \overline{U}$  is dense in  $\overline{U}$  for every  $Z \in \mathcal{Z}$ ;

- hence  $w(\overline{U}) \leq \mathfrak{c}$  by THM 4. ;

– hence  $w(X) \leq c$  using compactness and homogeneity.

 $-\pi\chi(X) = \omega$  as X is homogeneous;

- *X* has a cover  $\mathcal{H}$  by  $G_{\delta}$  sets of weight  $\leq \mathfrak{c}$ ;

 $- X = \cup \mathcal{Y}$  where  $|\mathcal{Y}| < \omega$  and every  $Y \in \mathcal{Y}$  is CT;

- there are  $\overline{U}$  regular closed and  $\mathcal{Z} \subset \mathcal{Y}$  such that  $\overline{U} \subset \cup \mathcal{Z}$  and  $Z \cap \overline{U}$  is dense in  $\overline{U}$  for every  $Z \in \mathcal{Z}$ ;

- hence  $w(\overline{U}) \leq \mathfrak{c}$  by THM 4. ;

– hence  $w(X) \leq c$  using compactness and homogeneity.

### On $\sigma$ -CT products

æ

イロト イヨト イヨト イヨト

## On $\sigma$ -CT products

What can we say if  $X^{\omega}$  is (also)  $\sigma$ -CT?

3

(I) < ((()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) <

#### THEOREM 5.

If the product  $X = \prod \{X_i : i \in I\}$  is  $\sigma$ -CT then all but finitely many of its factors  $X_i$  are CT.

A .

#### THEOREM 5.

If the product  $X = \prod \{X_i : i \in I\}$  is  $\sigma$ -CT then all but finitely many of its factors  $X_i$  are CT.

So, if  $X^{\omega}$  is  $\sigma$ -CT then X is CT.

- **→ → →** 

A .

#### THEOREM 5.

If the product  $X = \prod \{X_i : i \in I\}$  is  $\sigma$ -CT then all but finitely many of its factors  $X_i$  are CT.

So, if  $X^{\omega}$  is  $\sigma$ -CT then X is CT.

#### THANK YOU FOR YOUR ATTENTION !

- **→ → →** 

#### THEOREM 5.

If the product  $X = \prod \{X_i : i \in I\}$  is  $\sigma$ -CT then all but finitely many of its factors  $X_i$  are CT.

So, if  $X^{\omega}$  is  $\sigma$ -CT then X is CT.

#### THANK YOU FOR YOUR ATTENTION !

MANY THANKS TO THE ORGANIZERS!

### 6TH EUROPEAN SET THEORY CONFERENCE

#### JULY 3-7, 2017

#### MTA RÉNYI INSTITUTE OF MATHEMATICS, BUDAPEST

www.renyi.hu

István Juhász (MTA Rényi Institute)

A B F A B F

A .