

**CADGME 2012**

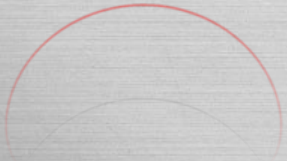
Novi Sad 22-24 June 2012

# **Correlation of GeoGebra and CAD software in the analysis of cycloid meshing**

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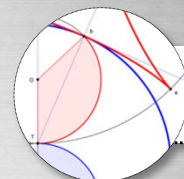
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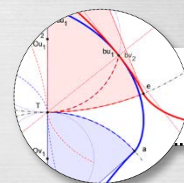
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4



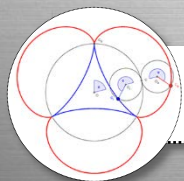
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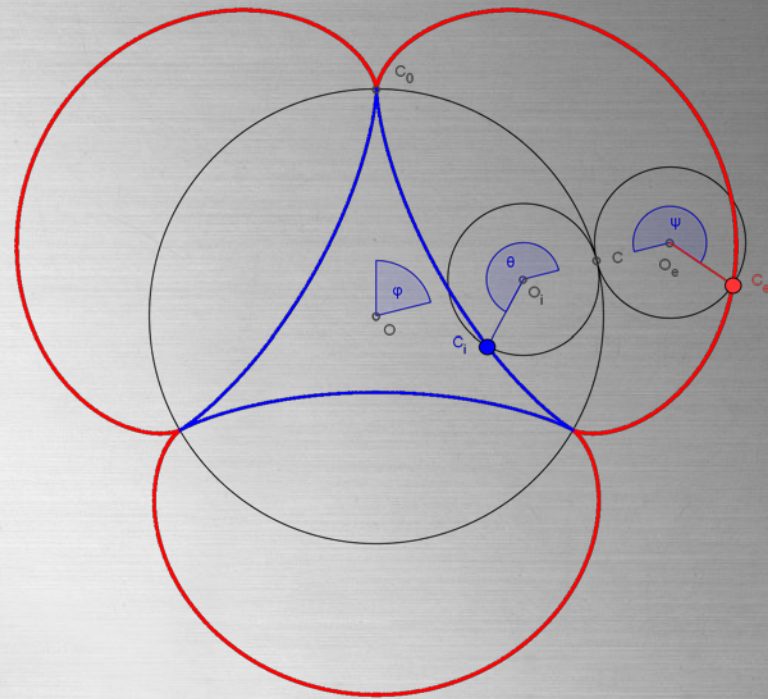
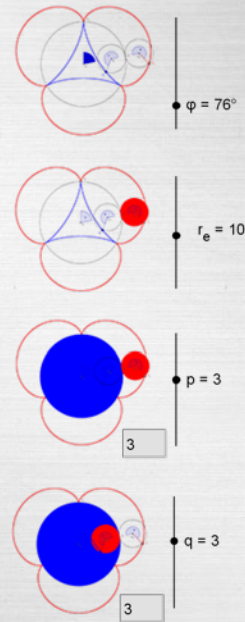


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...: Problems in engineering education  
...: Innovations in engineering education



- ☒ Epicycloid
- ☒ Hypocycloid
- ☐ Hypotrochoid
- ☐ Epitrochoid

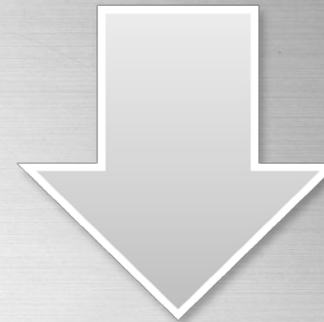
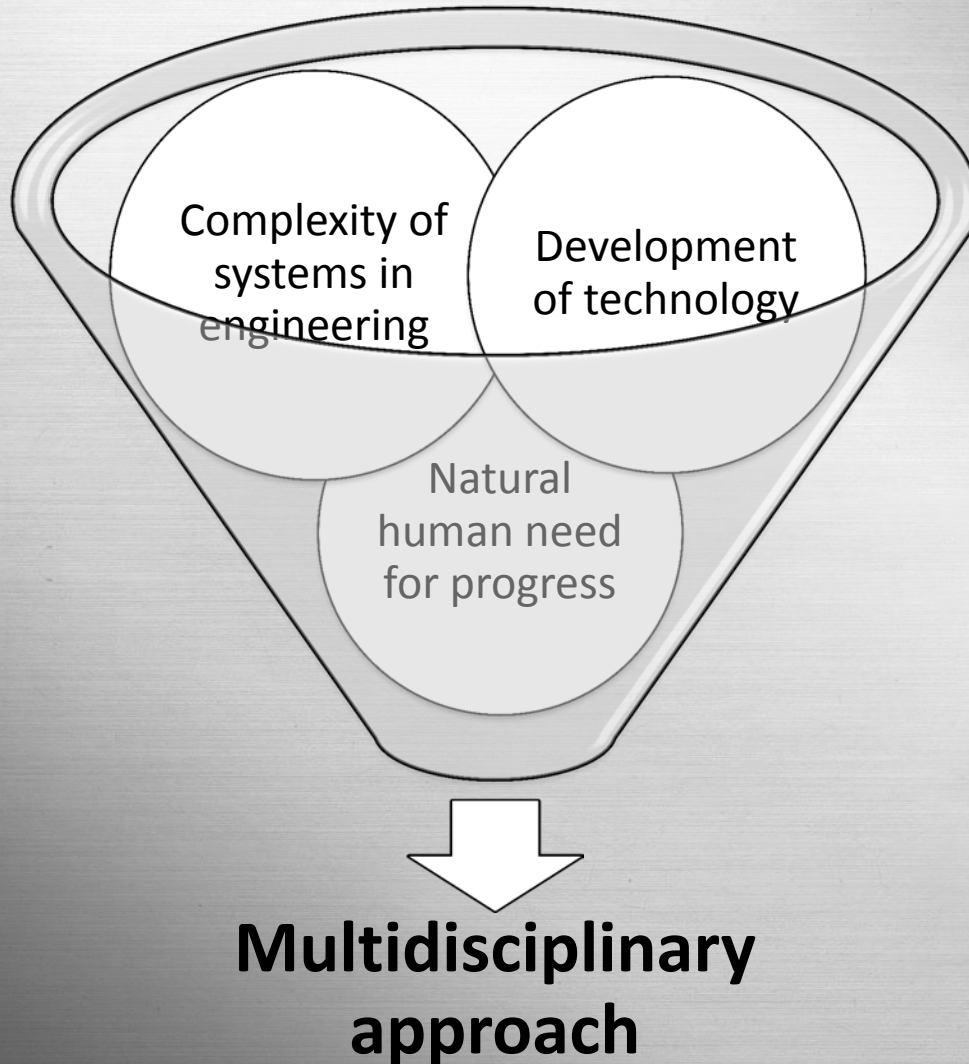
Part 1

# INTRODUCTION



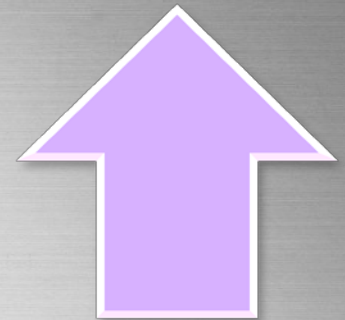
# Introduction

## Problems in engineering education



Low interest for studying engineering

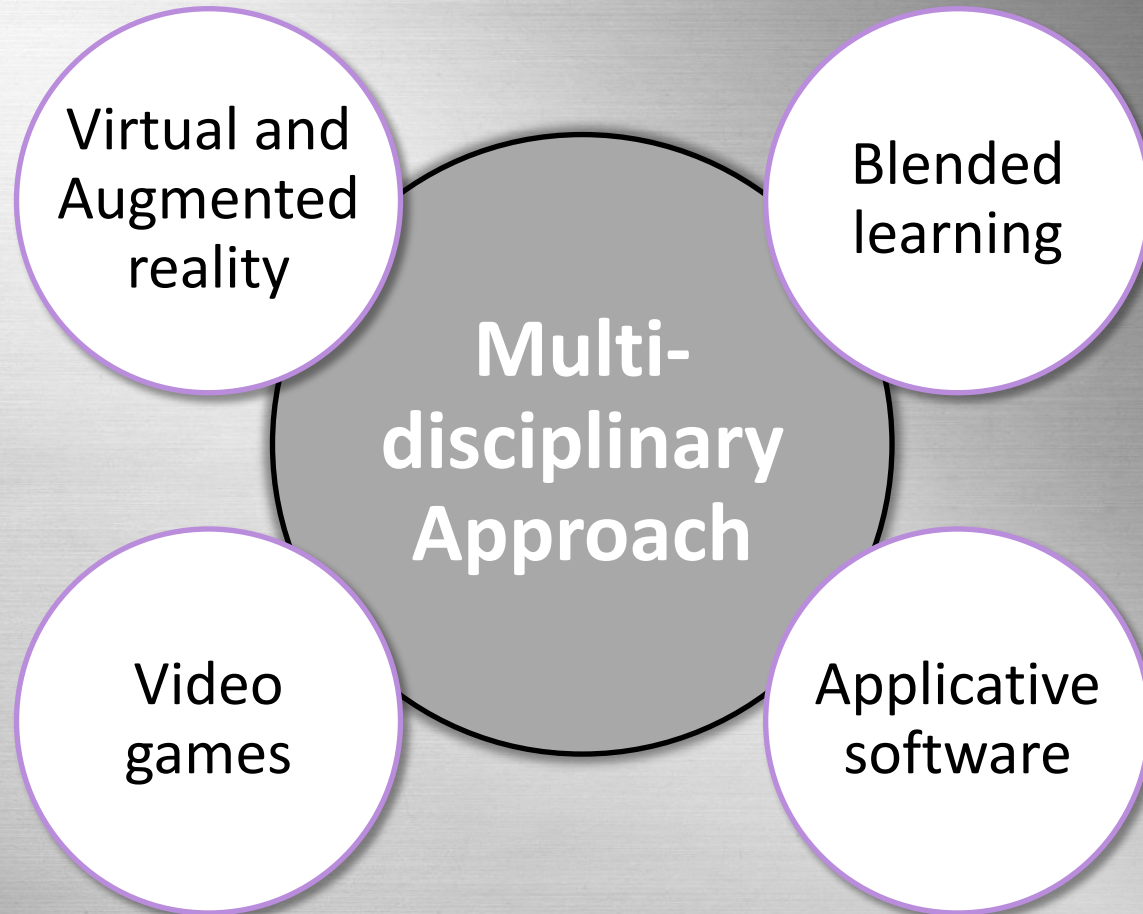
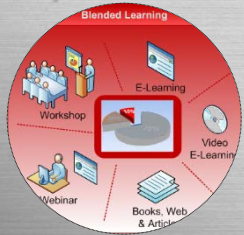
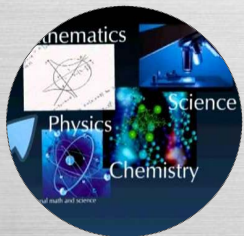
Development of new methods in education





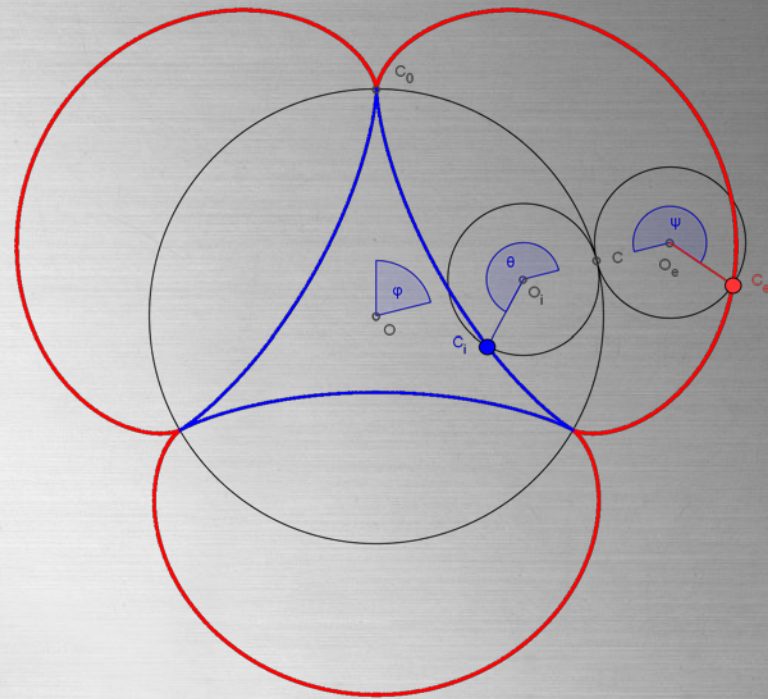
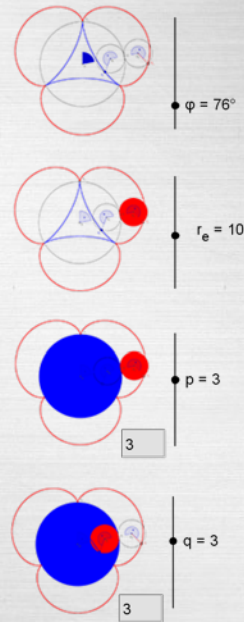
# Introduction

## Innovations in engineering education





- ∴ Motivation
- ∴ Basic knowledge
  - Theory of mechanisms
  - Basic Math
  - Application software



- ☒ Epicycloid
- ☒ Hypocycloid
- ☐ Hypotrochoid
- ☐ Epitrochoid

Part 2

# THE PROBLEM



# The problem

## Motivation



- **Actuality of the graduation papers**
  - Higher complexity of the assignments
  - New problems
  - Problems from real world
- **Introducing new teaching methods**
  - Multidisciplinary approach
  - Efficiency of acquainted knowledge
- **Why cycloid meshing?**
  - Frequently applied in practice
  - Basic theoretical experiment
  - Richness of different solutions
  - Complex example
  - Adequate with non classical methods of learning
  - Suitable for analysis in GeoGebra laboratory
  - Demanding CAD model

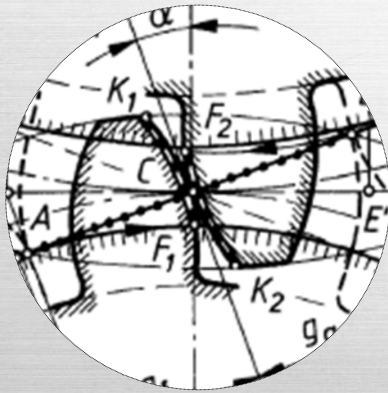


The 18-by-10-foot multimedia wall in the Collaborative Visualization Environment (CoVE) will be used in the new Professional Master's Degree in Applied Systems Engineering



# The problem

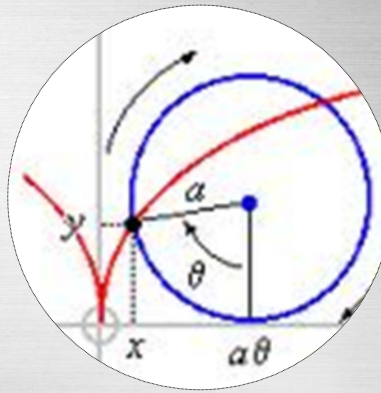
## Basic knowledge



### Theory of mechanisms

Pedagogical view

Basic theory

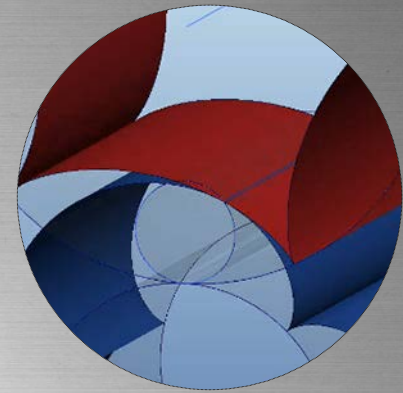


### Basic Math

Trigonometry

Analytic geometry in plane

Cycloids



### Application software

Math

CAD



# The problem

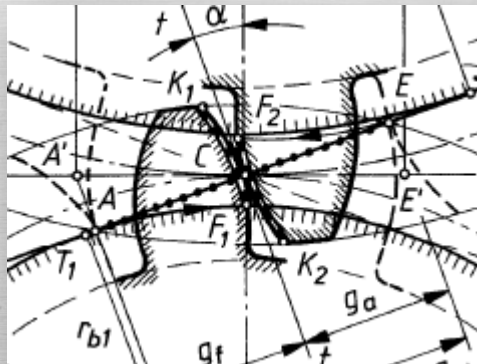
## Theory of mechanisms



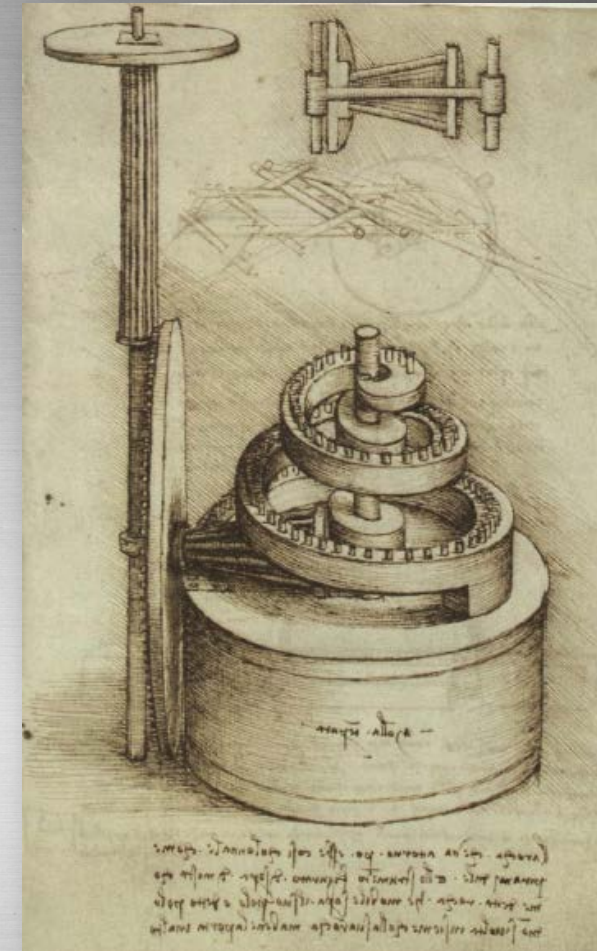
- Pedagogical view
  - Part of few core subjects of study during all four years
  - Students usually have learning problems
  - Teachers have teaching difficulties
  - Long and rich history
  - Science literature is not adequate for secondary level
  - School books are not enough



Antikythera mechanism,  
early 1st century BC



Roloff Matek  
Maschinenelemente, 2007



Leonardo da Vinci  
Madrid Codices I, 1493



# The problem

## Theory of mechanisms



### Basic theory

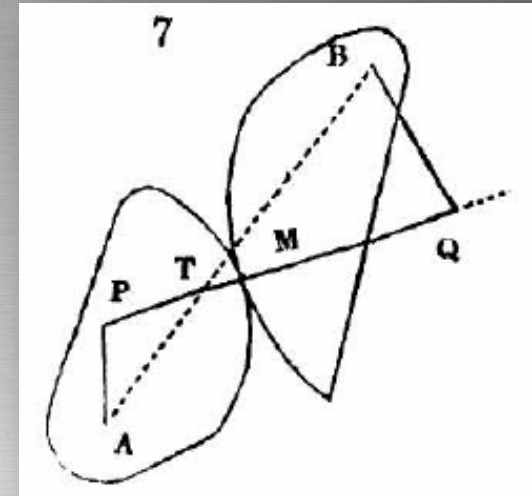
#### *Communication of Motion by Contact*

##### 1. *In the communication of motion by contact*

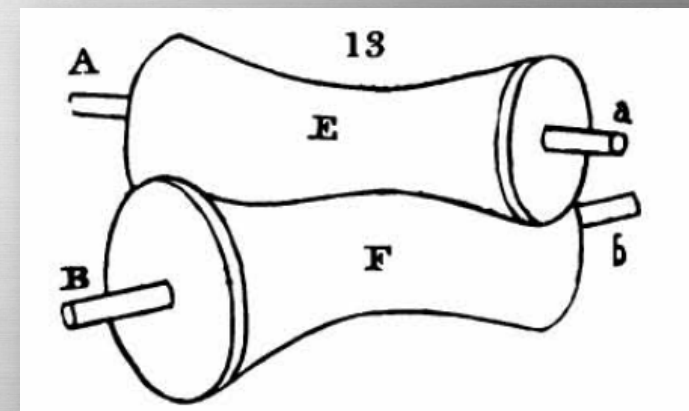
- the angular motions of the pieces are inversely as the segments into which the common normal divides the line of centers

##### 2. *In rolling contact*

- the point of contact is always in the line of centers
- the angular velocities are inversely as the segments into which the point of contact divides the line of centers
- *if the velocity ratio is constant*, the segments must be constant, and the curves become circles, revolving round their centers, and whose radii are the segments



*by Sliding Contact*



*by Rolling Contact*

Willis, Principles of Mechanism, 1841.



# The problem

## Theory of mechanisms

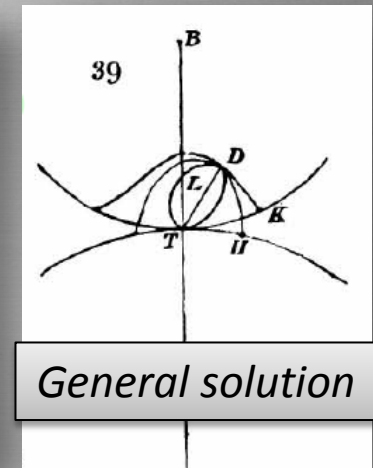
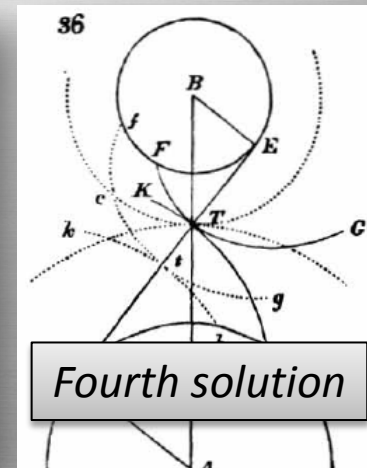
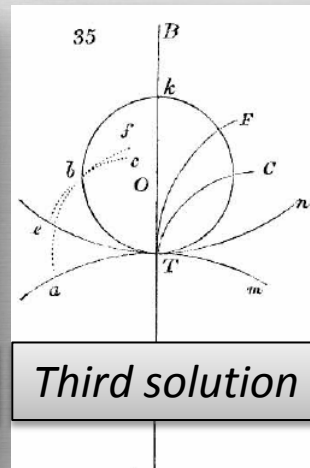
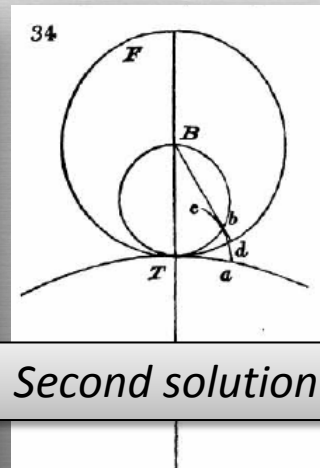
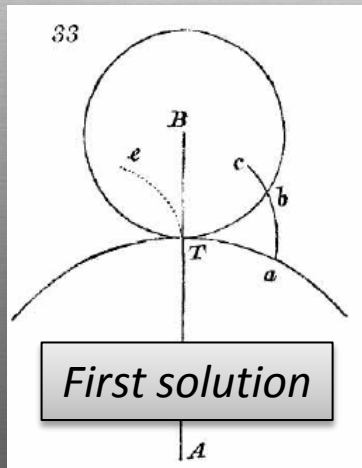


### Basic theory

Willis, Principles of Mechanism, 1841.

### *Communication of Motion by Sliding Contact*

1. The axes being supposed parallel
2. The angular velocities are in the inverse ratio of the segments into which the normal of the curves, at the point of contact, divides the line of centers
3. Any curve then being assumed for the edge of one revolving piece
4. If we can assign such a form to the edge of another revolving piece that the common normal of the two curves shall divide the line of centers in a fixed point, in all positions of contact, then will these curves preserve a **constant angular velocity ratio**





# The problem

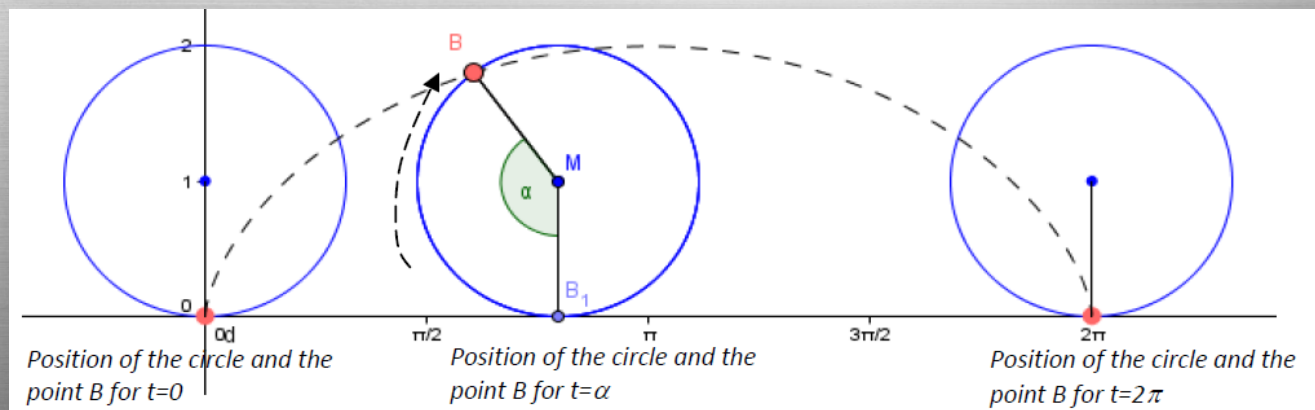
## Basic Math



- Trigonometry, Analytic geometry in plane
- Both are parts of the curriculum
- Cycloid

$$\begin{aligned}x &= r(t - \sin t) \\ y &= r(1 - \cos t)\end{aligned}$$

- Although, parametric equations of the curves are not part of the curriculum by the use of GeoGebra students except them easily
- It is known that the so-called mechanical curves are not the subject of study at the secondary level. However, by the use of GeoGebra, analysis of these curves is qualitatively possible and acceptable for students



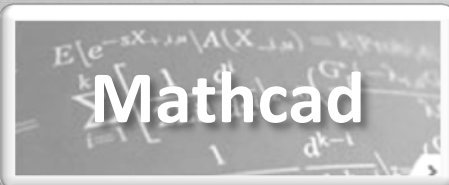


# The problem

## Application software



GeoGebra is an interactive geometry, algebra, and calculus application, intended for teachers and students



Mathcad is computer software primarily intended for the verification, validation, documentation and re-use of engineering calculations

**Math software**

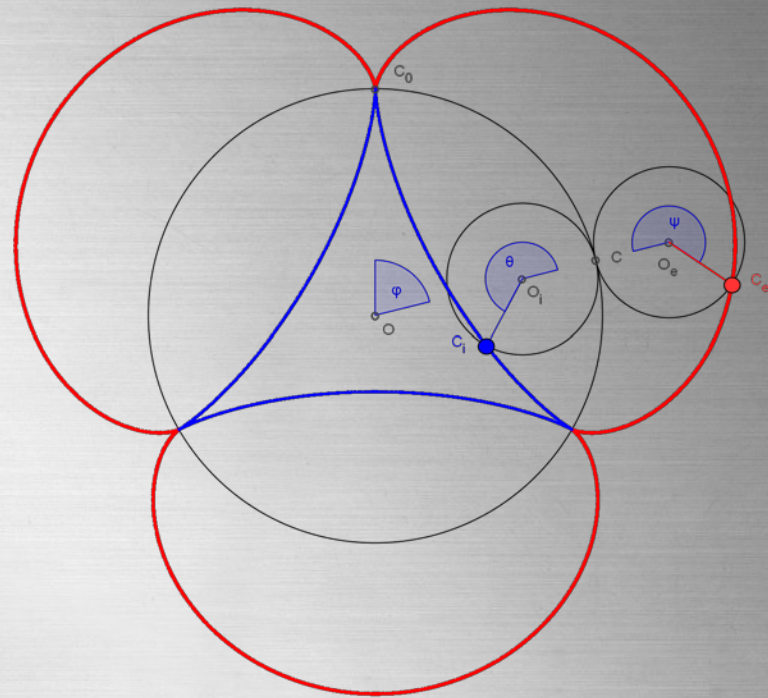
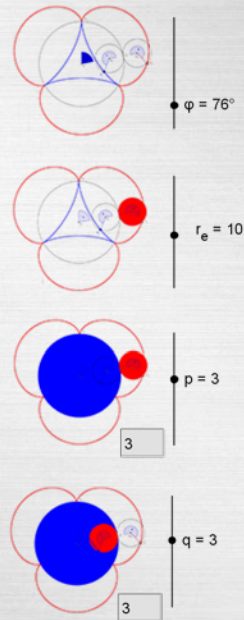


Creo Elements/Pro (formerly Pro/ENGINEER) parametric, integrated 3D CAD/CAM/CAE solution for design and manufacturing

**CAD software**



...: with GeoGebra  
...: Examples  
...: Conclusion



- ☒ Epicycloid
- ☒ Hypocycloid
- ☐ Hypotrochoid
- ☐ Epitrochoid

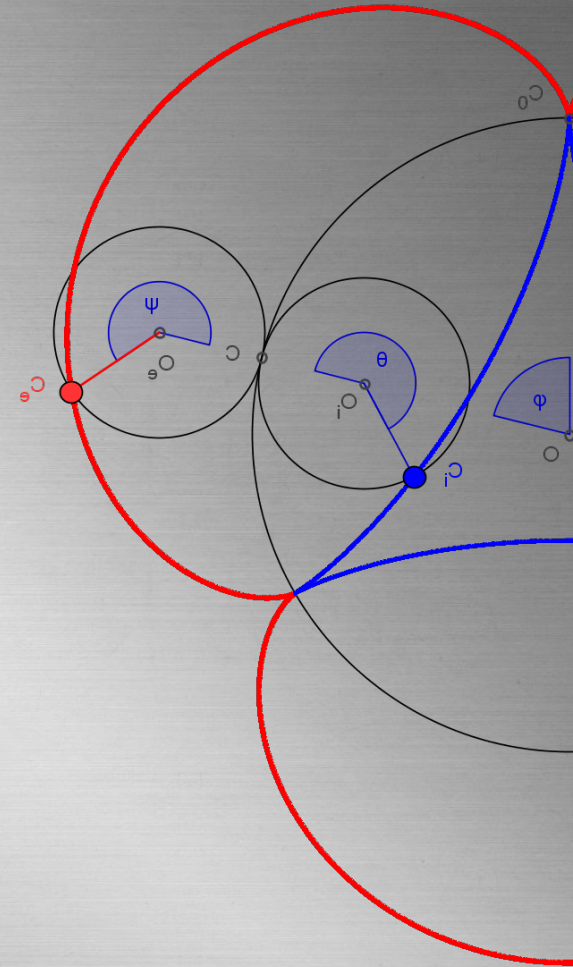
Part 3

# CYCLOID LABORATORY



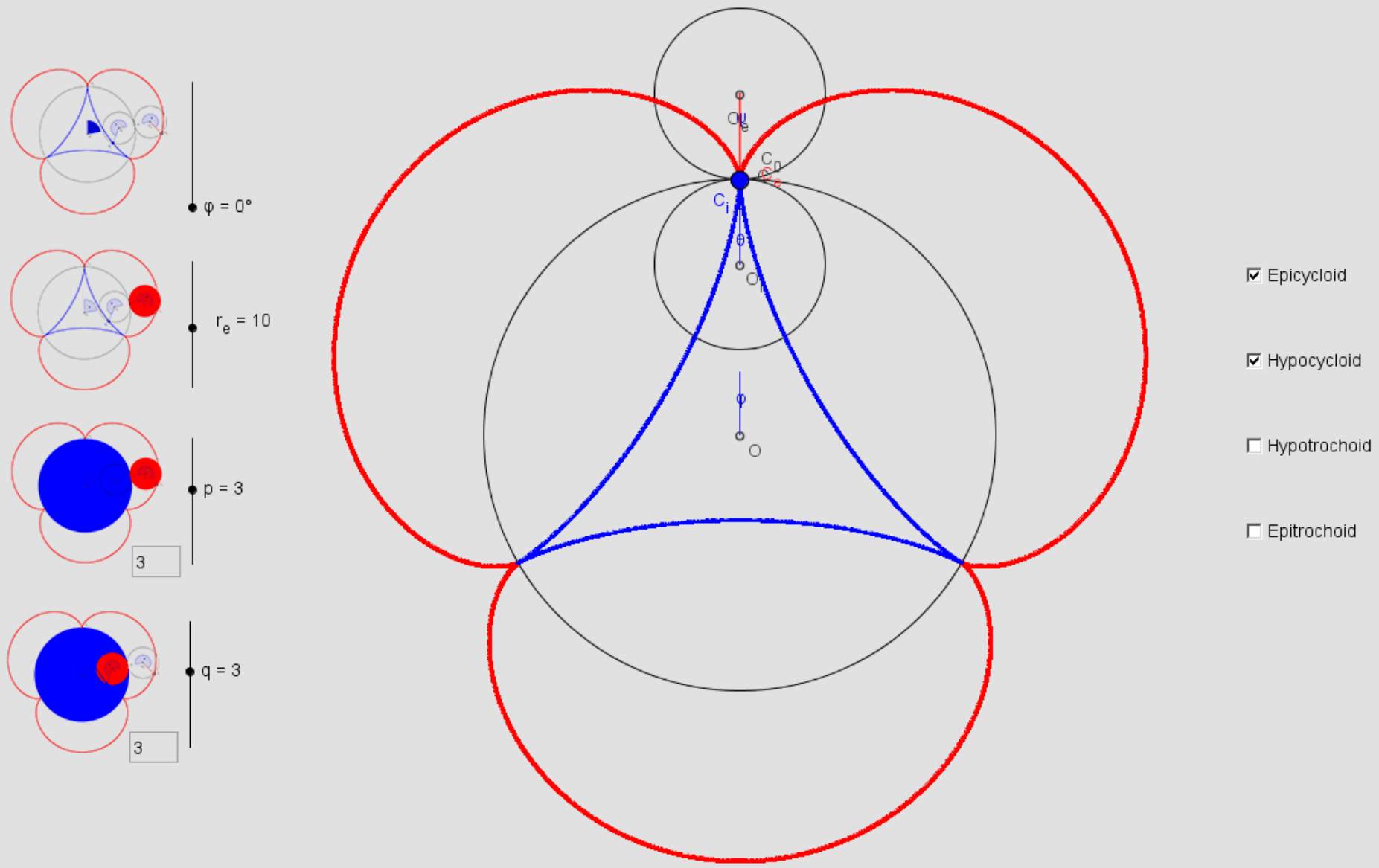


- When a straight line rolls along a stationary circle a point on the line traces a curve called an **involute**
- When a circle rolls along a stationary straight line a point on the circumference of the circle traces a curve called a **cycloid**
- When a circle rolls along another circle then a point on the circumference of the rolling circle traces out a curve called an **epicycloid** (if the rolling circle rolls on the outside of the stationary circle)  
or a **hypocycloid** (if the rolling circle rolls on the inside of the stationary circle).
- In all these cases of rolling circles points not on the circumference trace curves called **trochoids**



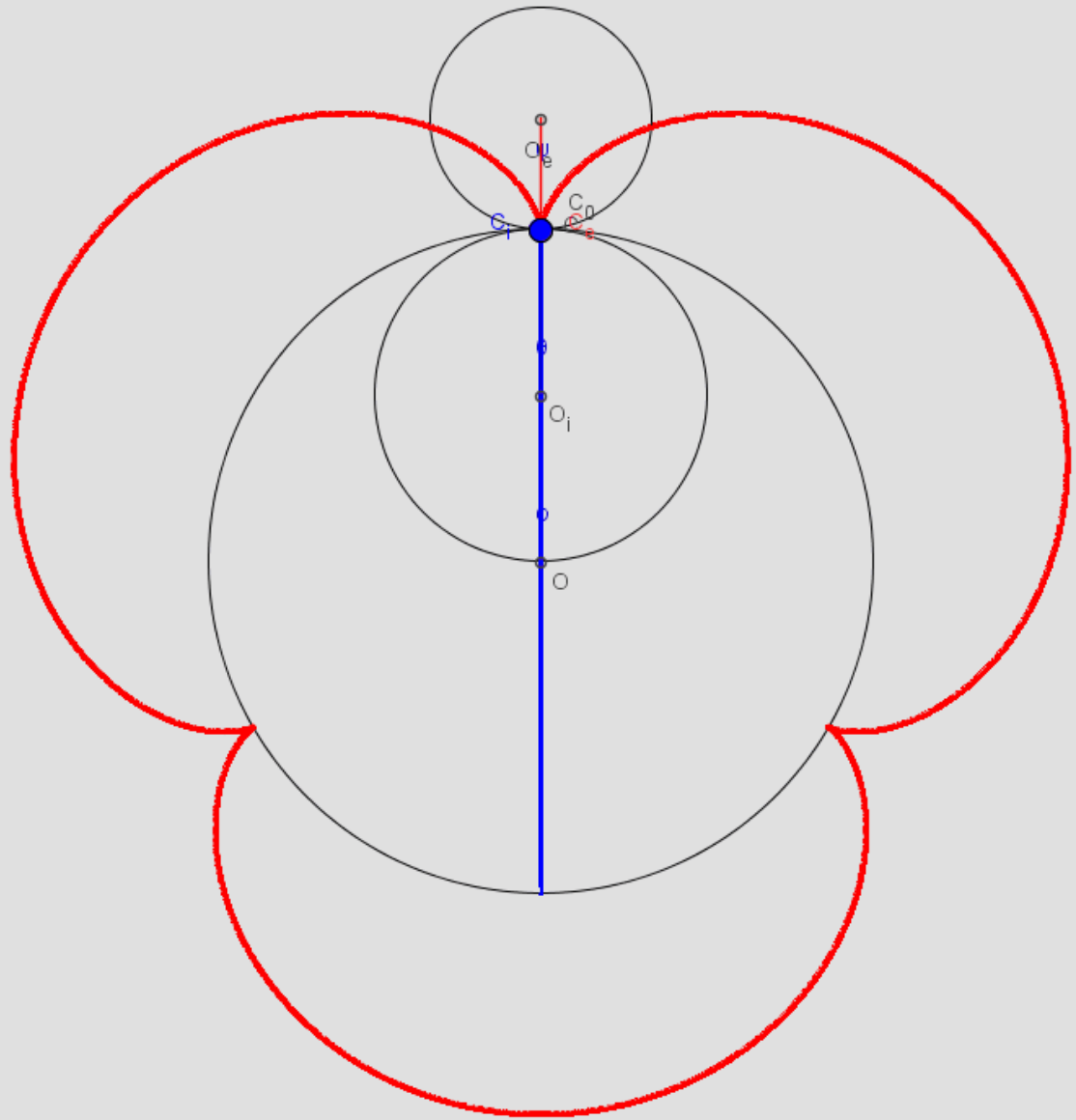
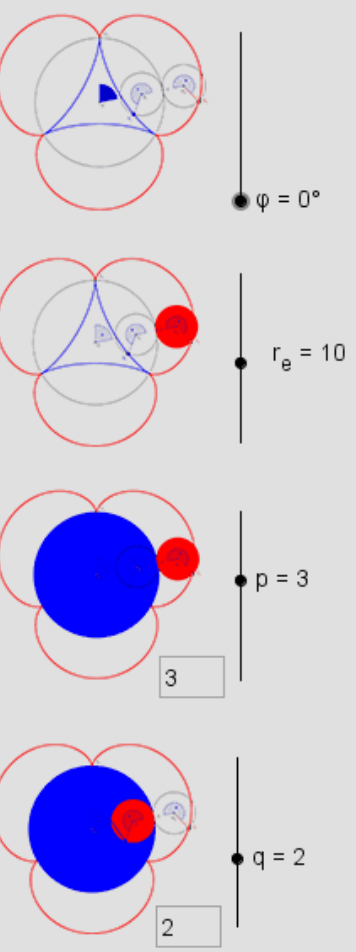


# Example 1





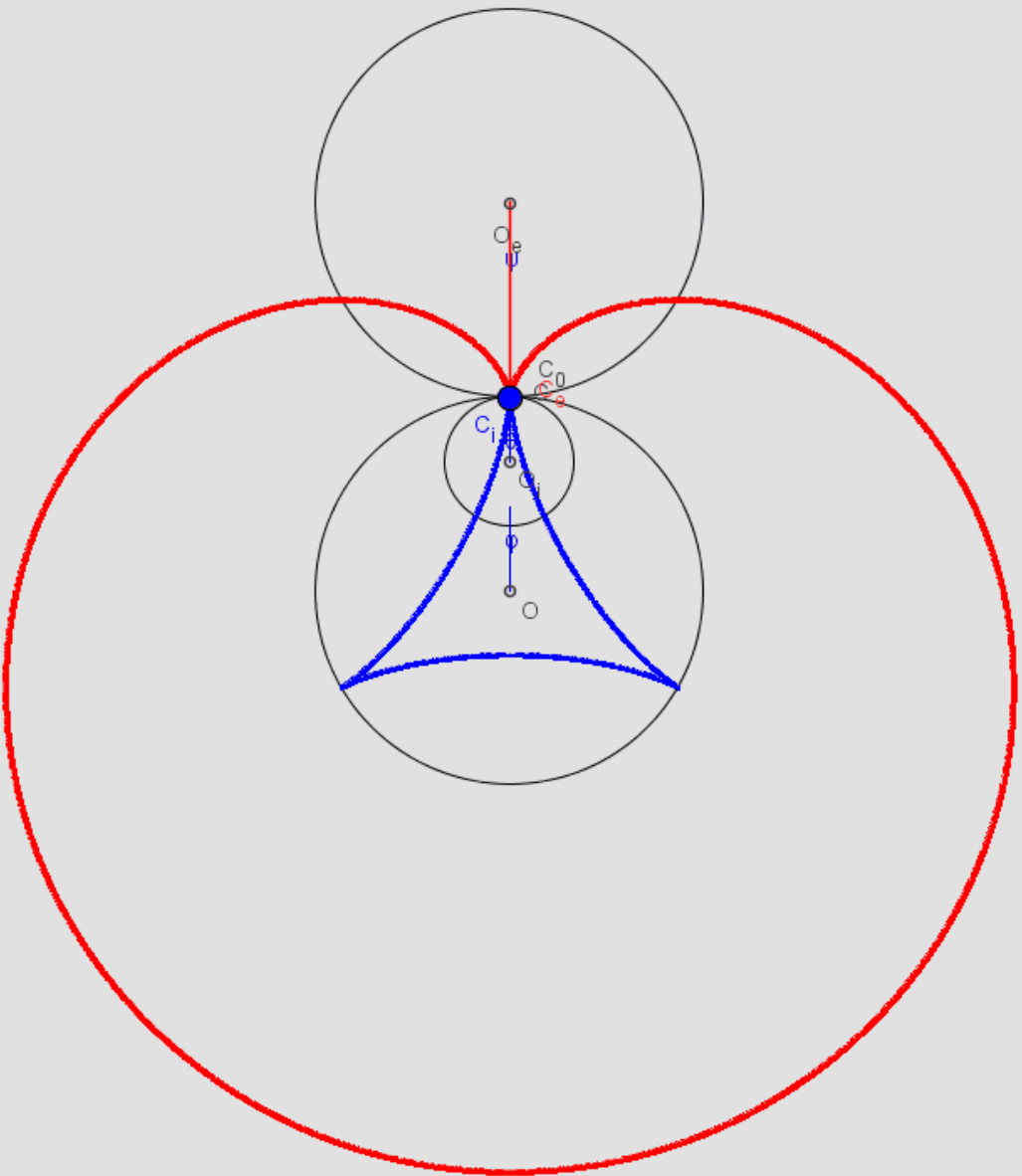
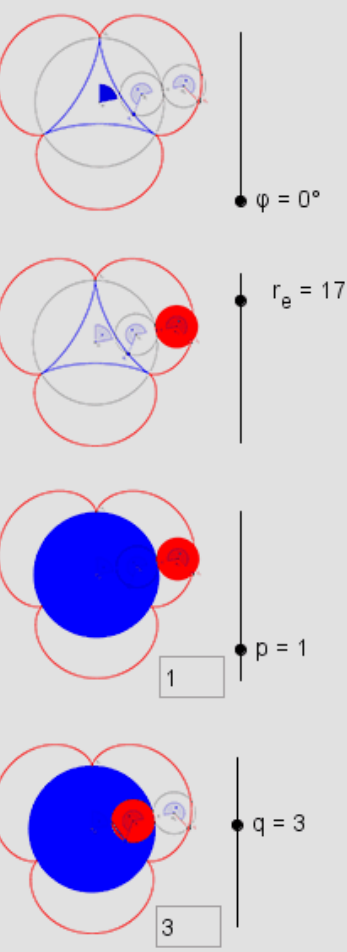
# Example 2



- ☒ Epicycloid
- ☒ Hypocycloid
- ☐ Hypotrochoid
- ☐ Epitrochoid



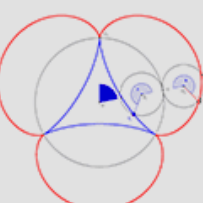
Example 3



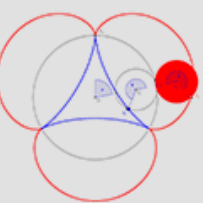
- ☒ Epicycloid
- ☒ Hypocycloid
- ☐ Hypotrochoid
- ☐ Epitrochoid



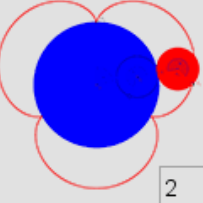
Example 4



$\varphi = 0^\circ$

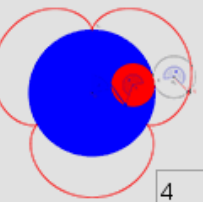


$r_e = 13$



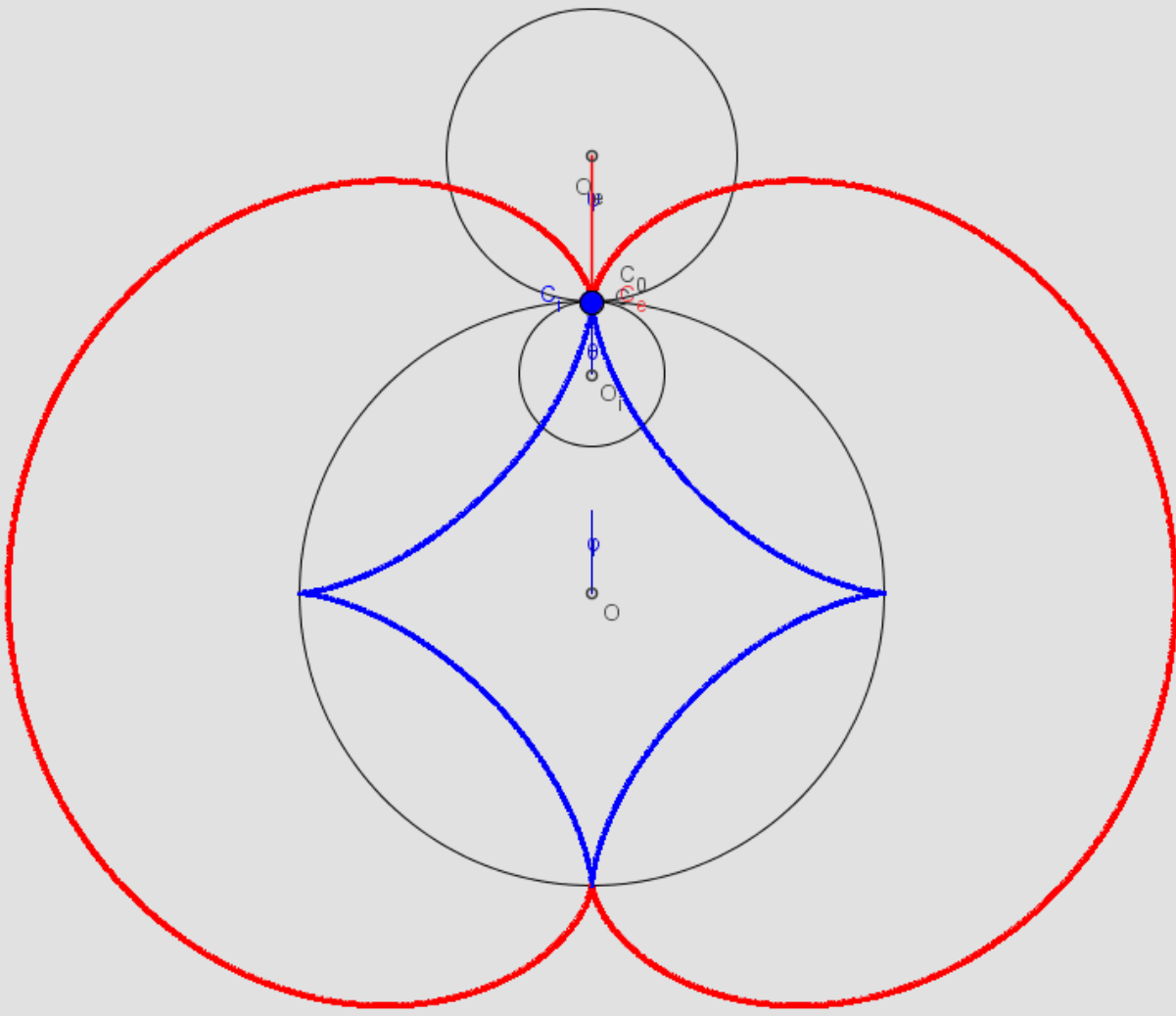
$p = 2$

2



$q = 4$

4



- ☒ Epicycloid
- ☒ Hypocycloid
- ☐ Hypotrochoid
- ☐ Epitrochoid



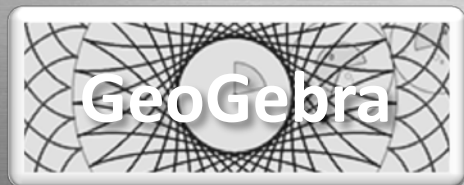


- The aim of GeoGebra laboratory is for student to
  - To spot kinematic connections
  - To spot trajectory of the fixed point
  - To grasp the mining of the ratio
  - To grasp the correlation between the given ratio and the form of the curve
  - To grasp the mining of proportions being integers





- The aim of GeoGebra laboratory is for student to
  - To spot kinematic connections
  - To spot trajectory of the fixed point
  - To grasp the mining of the proportions
  - To grasp the correlation between the proportions and the form of the curve
  - To grasp the mining of proportions being integers



Mathcad

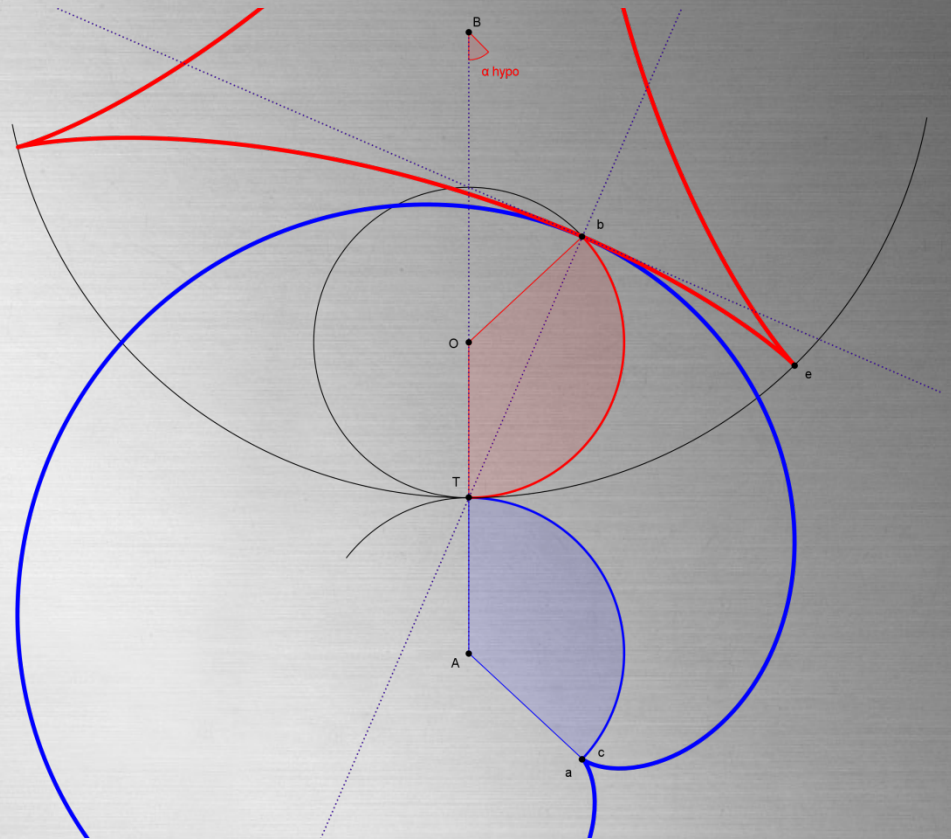




- ...: First Solution
- ...: Second Solution
- ...: Third Solution
- ...: Application

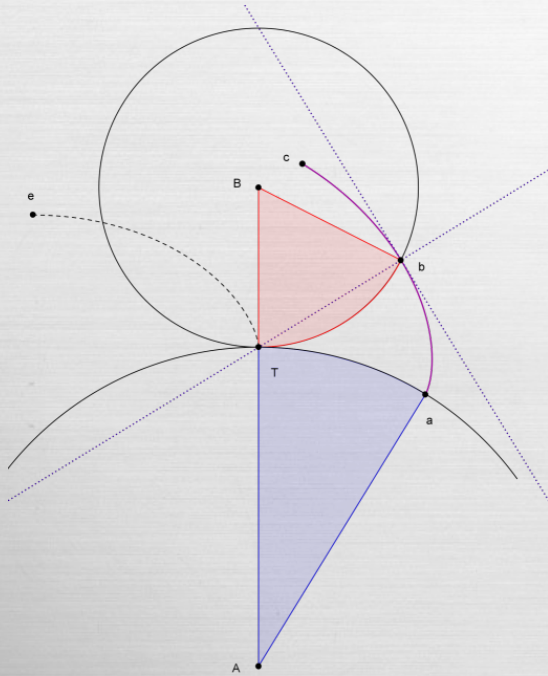
Part 4

# CYCLOID MESHING

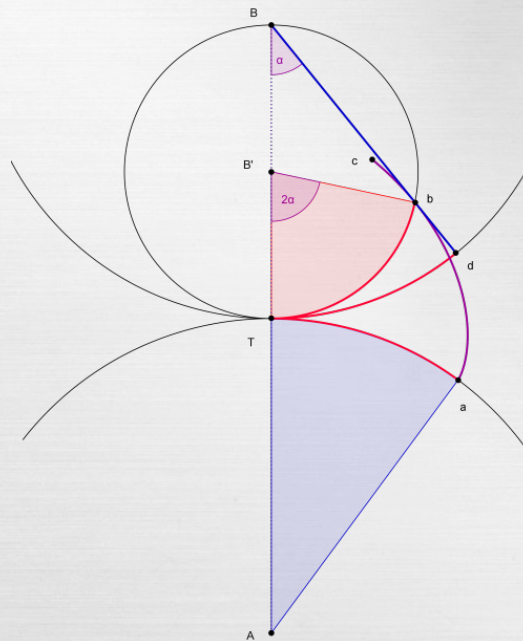




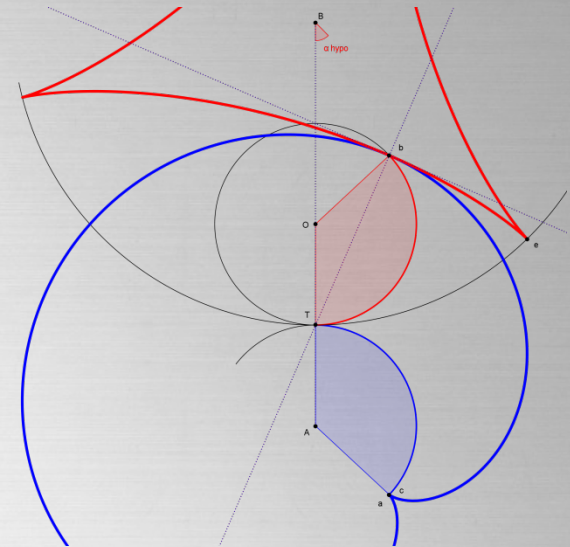
# Cycloid meshing



First Solution



Second Solution



Third Solution

Willis, Principles of Mechanism, 1841.



# First Solution

$$\beta = 0^\circ$$

$$r_1 = 10$$

$$r_2 = 5$$

☒ Comment 1

☒ Comment 2

☒ Comment 3

Let A, B be the centers of motion, AB the line of centers divided as usual, in T, in the inverse proportion of the angular velocities; describe through T the respective pitch circles, and let abc be a portion of an epicycloid whose base is the pitch circle aT, and whose describing circle has the same diameter as the pitch circle Tb, and let b a pin whose diameter is exceedingly small, so that it may be considered as a mathematical line. Then if the curve abc be cut out of a thin plate, and caused to turn round the center A, and the pin b carried by a piece capable of turning round the center B, the motion communicated from the edge to the pin will fulfil the required conditions.

If the curve move into any other position abc driving the pin to b, the arc Ta will be equal to Tb; for Tb is an arc of the describing circle, and therefore, if it were made to roll on Ta, the point b would trace an epicycloidal arc coinciding with ba; and the point b would coincide with a. But the arcs Ta, Tb are also those described by the two pitch circles respectively, in moving from T to the second position;

And since these equal arcs are described in the same time, the angular velocity ratio of the two pieces is constant, and the same as if the motion had been produced by the rolling contact of the pitch circle.

The axes being supposed parallel, it appears, that in sliding contact, the angular velocities are in the inverse ratio of the segments into which the normal of the curves, at the point of contact, divides the line of centers.

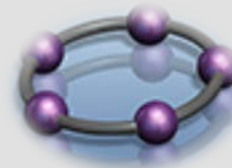
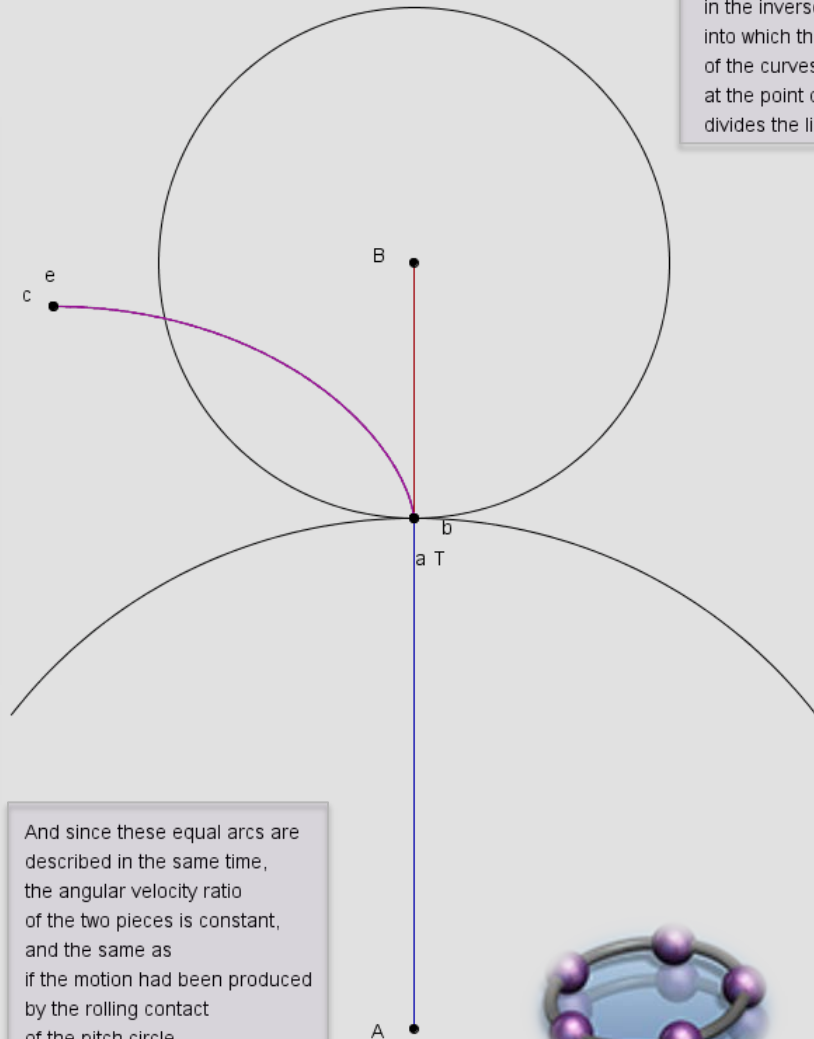
The common normal of the two curves shall divide the line of centers in a fixed point, in all positions of contact, then will these curves preserve a constant angular velocity ratio.

By the known property of the epicycloid, the normal to any point b passes through the point of contingency T of its describing circle and its base circle.

The angular velocity ratio of the circles will be constant and equal to the inverse ratio of their radii

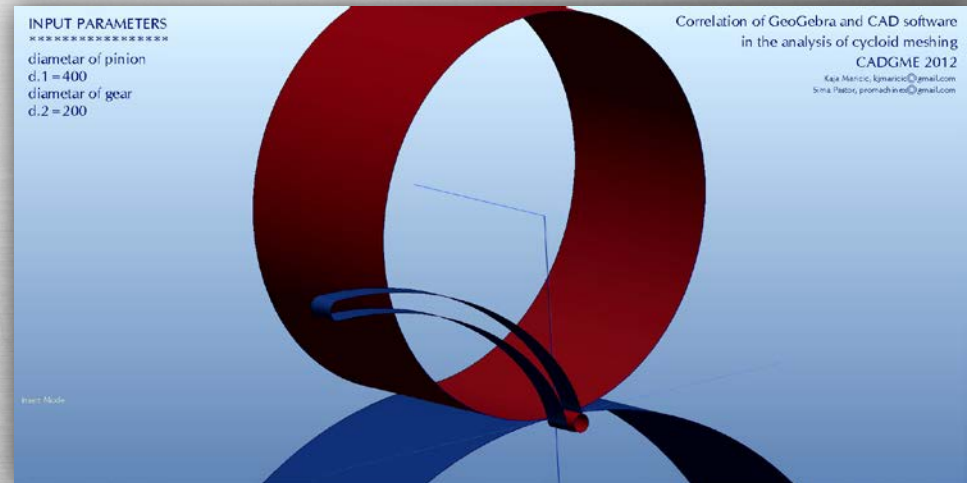
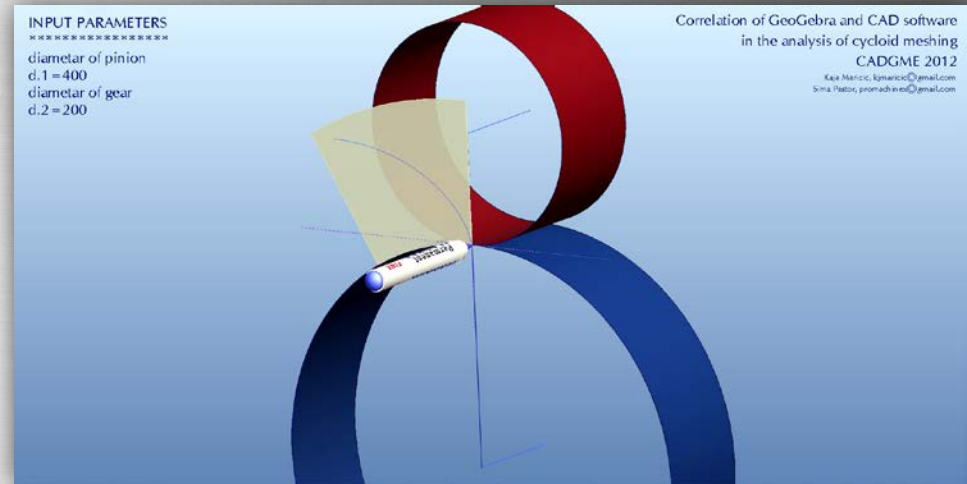
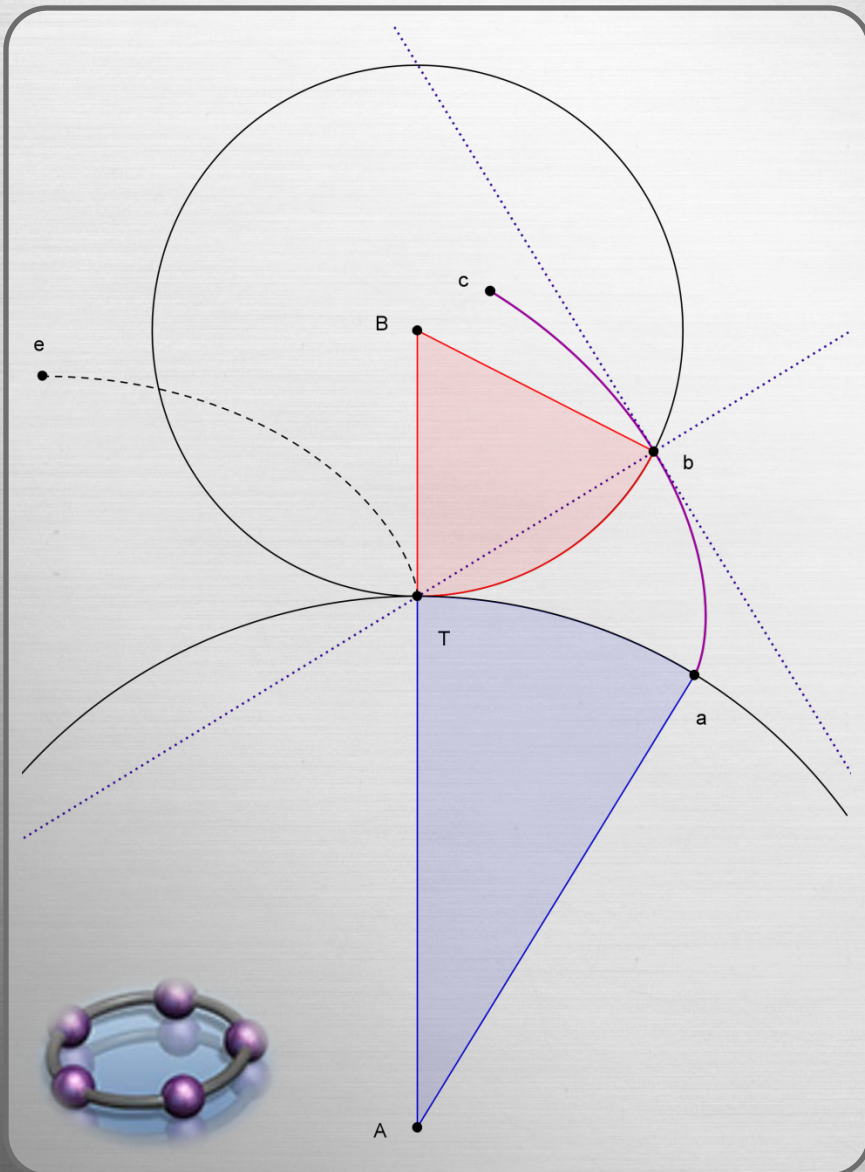
Otherwise, by the known property of the epicycloid, the normal to any point b passes through the point of contingency T of its describing circle and its base circle.

But these latter circles are the two pitch circles of the combination; and since the normal of the curve ab at the point of the contact is thus shown to pass through a constant point T of the line of centers, the angular velocity ratio of the circles will be constant and equal to the inverse ratio of their radii.





# First Solution





# Second Solution

$$\beta = 0^\circ$$

$$r1 = 15$$

$$r2 = 7$$

✓ Comment 1

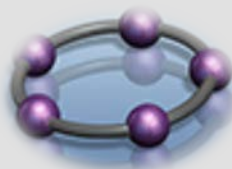
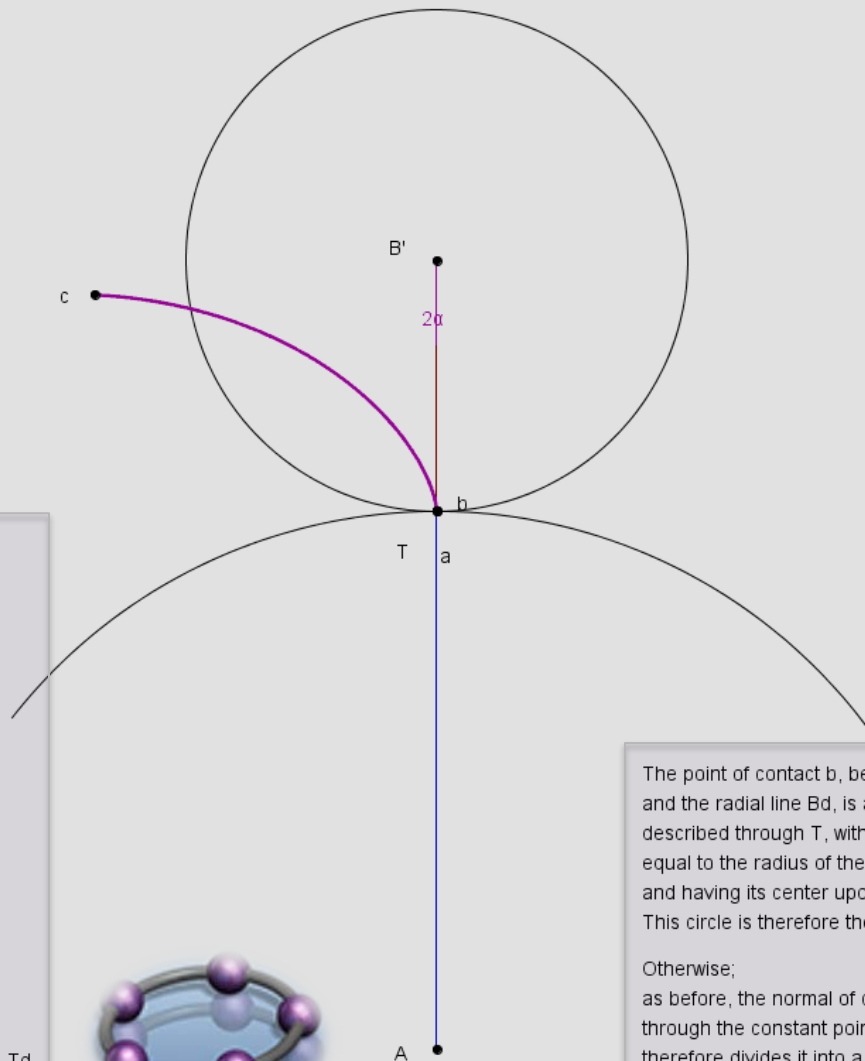
✓ Comment 2

✓ Comment 3

Let  $abc$  be an arc of an epicycloid whose describing circle is  $TbB$ , of half the diameter of the pitch circle  $TdF$ . From the center  $B$  draw a radial line through the describing point  $b$ , meeting the circle in  $d$ ; then will this line touch the epicycloid in  $b$ .

Let motion be communicated by contact from the curved edge  $abc$ , which revolves round  $A$ , to the radial line  $Bbd$ , which revolves round  $B$ ;

In moving to any other position of contact  $abc$ ,  $Bbd$ ;  $Ta$ ,  $Td$ , will be the arcs simultaneously described by the two pitch circles.



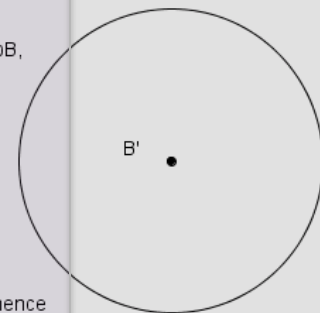
Now  $\alpha$  is an angle at the circumference of the circle  $TbB$ , and  $\alpha$  an angle at the center of the circle  $TdF$ ; therefore  $Tb$  measures an angle double of  $Td$ . Also the radius of  $Tb$  is half that of  $Td$ ; therefore the arc  $Tb = Td$ .

Again,  $TbB$  is the describing circle of the epicycloid  $abc$ , and  $Ta$  its base, therefore  $Tb = Ta$ .

whence  $Td = Ta$ , that is, the arcs of the pitch circle described from the beginning of the motion are equal, and consequently the angular velocity ratio constant, and the same as would be obtained by the rolling contact of the pitch circles.

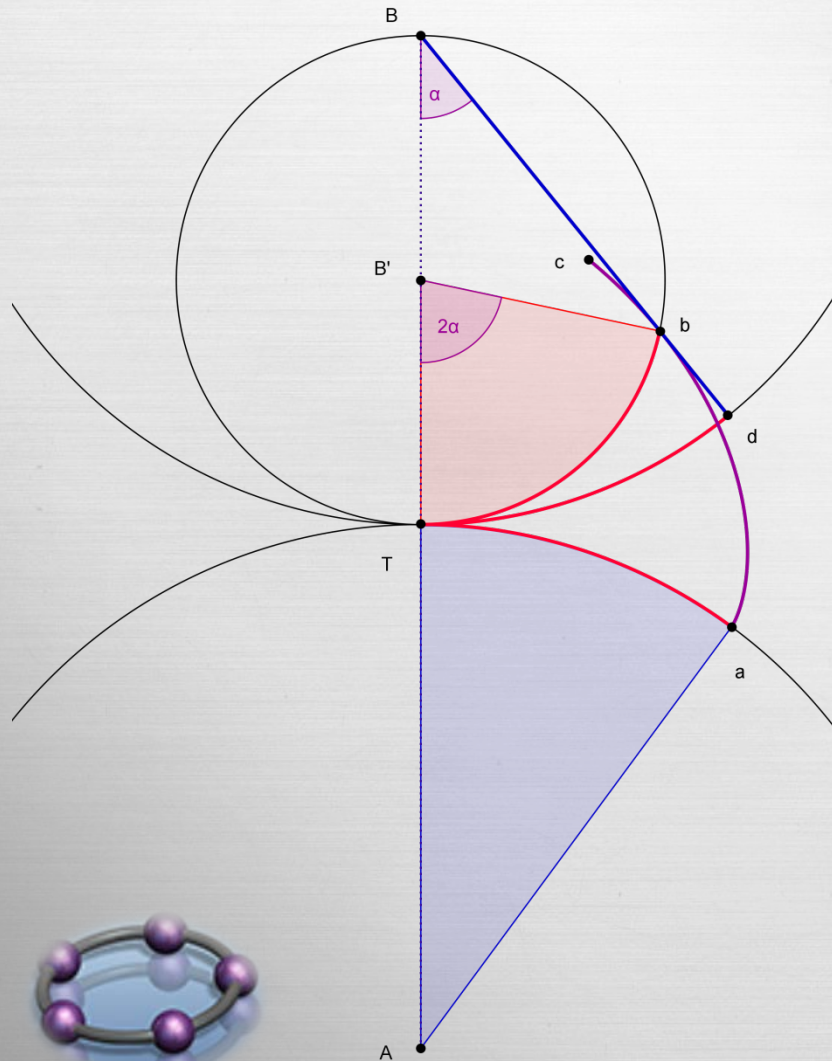
The point of contact  $b$ , between the curve  $ac$  and the radial line  $Bbd$ , is always situated in the circle  $TbB$ , described through  $T$ , with a diameter equal to the radius of the pitch circle of the radial line, and having its center upon the line of centers. This circle is therefore the locus of contact.

Otherwise; as before, the normal of contact at  $b$  passes through the constant point  $T$  of the line of centers, and therefore divides it into a pair of constant segments; whence by Art. 75 the angular velocity ratio is constant.



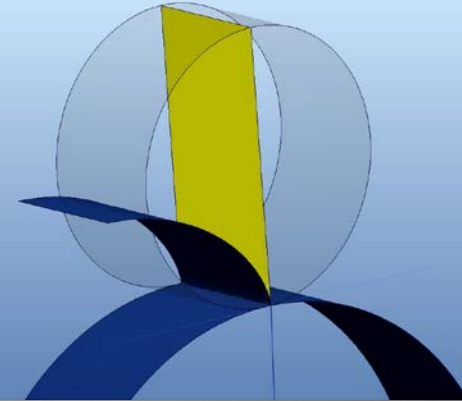


# Second Solution



INPUT PARAMETERS  
\*\*\*\*\*  
diameter of pinion  
 $d_1 = 400$   
diameter of gear  
 $d_2 = 200$

Correlation of GeoGebra and CAD software  
in the analysis of cycloid meshing  
CADGME 2012  
Kaja Maric, kmaric@gmail.com  
Sima Radoj, simaradoj@gmail.com





# Third Solution

$$\beta = 0^\circ$$

$$p = 4$$

$$q = 4$$

$$r_e = 13$$

✓ Comment 1

✓ Comment 2

But these are the arcs respectively described by the two pitch circles in moving from the first position to the second; therefore, as before, the angular velocity ratio is constant and equal to that which would be obtained by the rolling contact of the pitch circles.

Otherwise; as before, the constancy of the angular velocity ratio may be shown from the known property of the curves by which the normal from the point  $b$  passes through  $T$ .

This third solution includes the two former ones, for it is known that if the diameter of the describing circle of an hypocycloid be made equal to the radius of the base, the hypocycloid becomes a straight line coinciding with a diameter of the latter; and thus the second solution is obtained.

Also, if the describing circle of the hypocycloid equal the circle of the base, the hypocycloid is reduced to a point in its circumference, and thus the first solution is obtained.

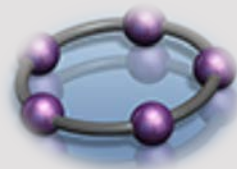
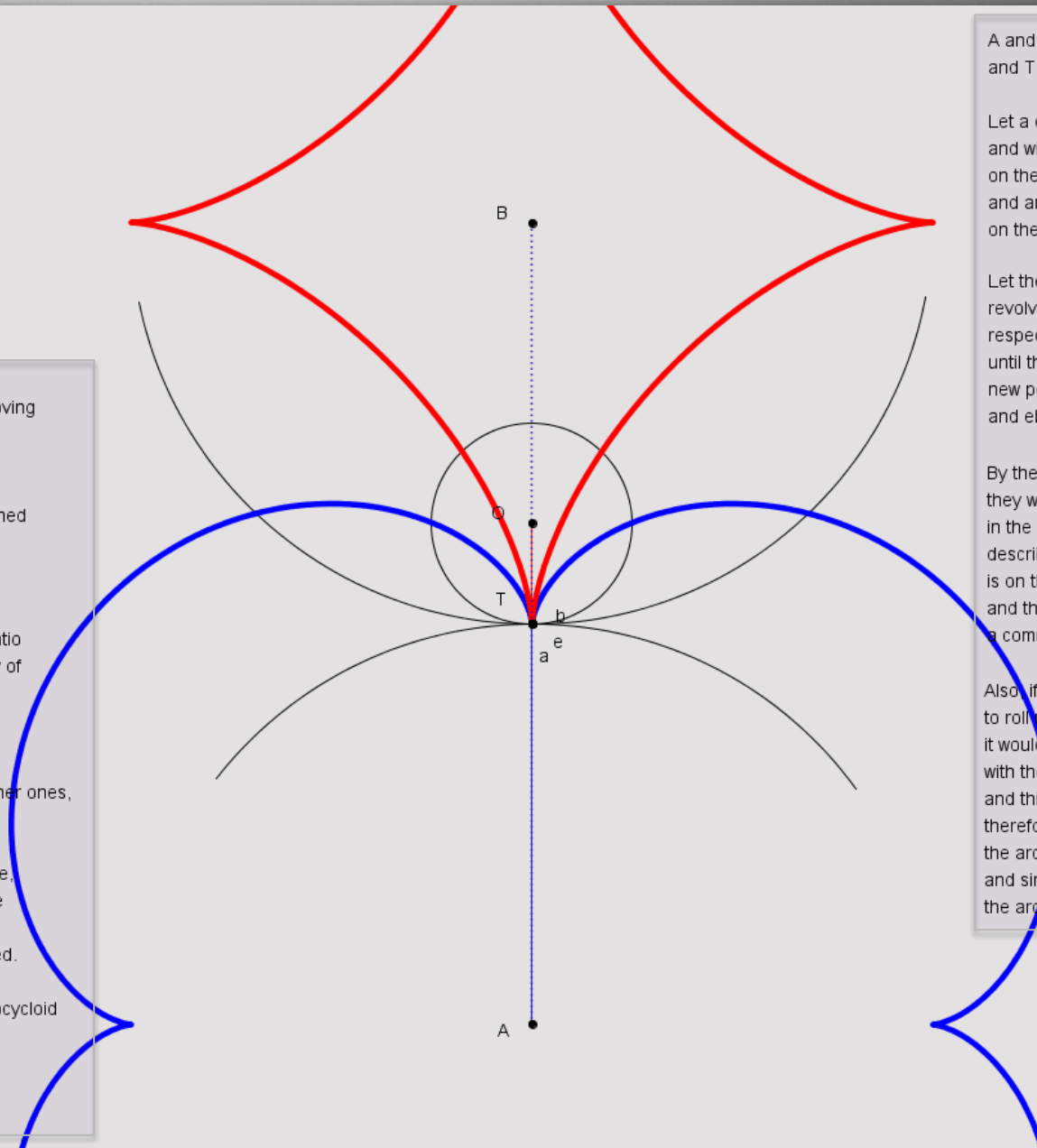
A and B being, as before, the centers of motion, and T the point of contingency of the pitch circles.

Let a describing circle  $r_e$  be taken of any diameter, and with it describe an epicycloid  $abc_0$  by rolling on the outside of the pitch circle  $r_0\text{epi}$ , and an hypocycloid  $ebf_0$  by rolling on the inside of the pitch circle  $r_0\text{hypo}$ .

Let these curves be cut out and made to revolve in contact, round their respective centers of motion A and B, until they come into a new position where  $abc$  is the epicycloid and  $ebf$  the hypocycloid.

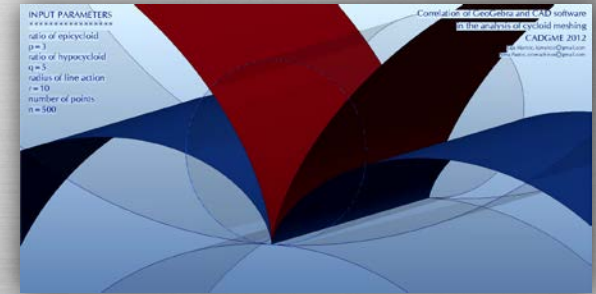
By the known properties of the curves they will have their common point  $b$  in the circumference of the describing circle  $r_e$ , when its center O is on the line of centers, and they will also have a common tangent there.

Also if the describing circle  $r_i$  were to roll upon  $r_0\text{hypo}$  from its present position, it would describe the curve  $be$  with the point  $b$ , and this point would come to  $e$ ; therefore the arc  $Tb$  is equal to the arc  $Te$ , and similarly, the arc  $Tb$  is equal to the arc  $Ta$ ;  $Te = Ta$ .





The diagram illustrates the formation of molecular orbitals from two atomic orbitals, A and B, in a diatomic molecule. The vertical axis represents the internuclear distance, with point T at the equilibrium bond length. Point O is the origin of the coordinate system. The shaded regions represent the bonding and antibonding molecular orbitals. The bonding orbital is shaded in light blue, and the antibonding orbital is shaded in light red. The points A, B, T, O, b, c, a, and e are labeled on the diagram. A small inset in the bottom left corner shows a molecular model of a diatomic molecule with two purple spheres representing the atoms and a blue ring representing the molecular orbital.

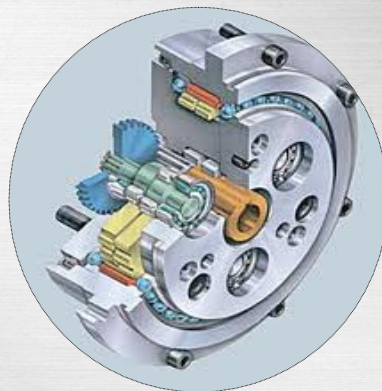




# Cycloid Meshing Application



Cycloidal drives



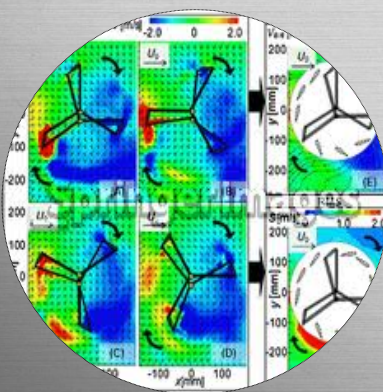
Cycloidal gears



Cycloid  
hydraulic motor



MEMS



Cycloidal propeller



Cycloidal flow meters



Wankel rotary engine

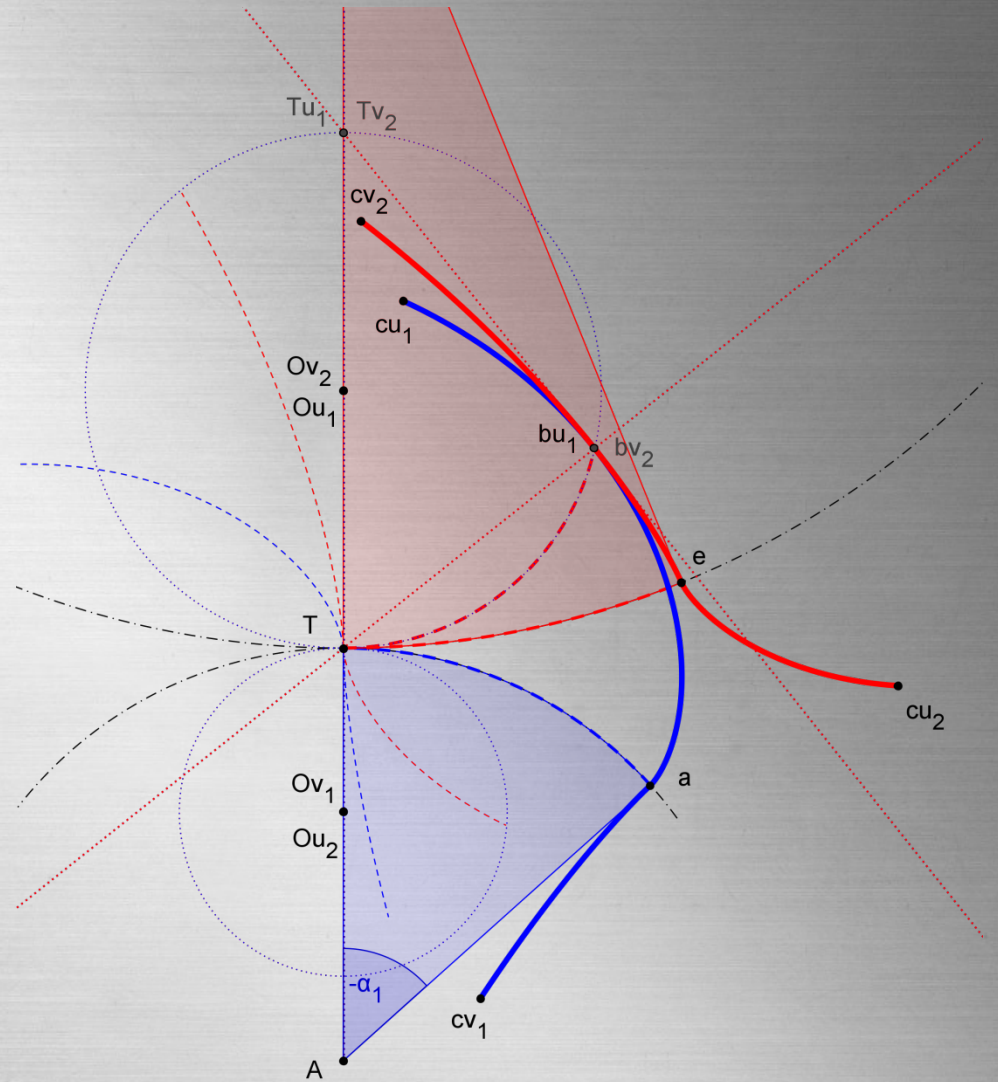


The Reuleaux polygon



Part 5

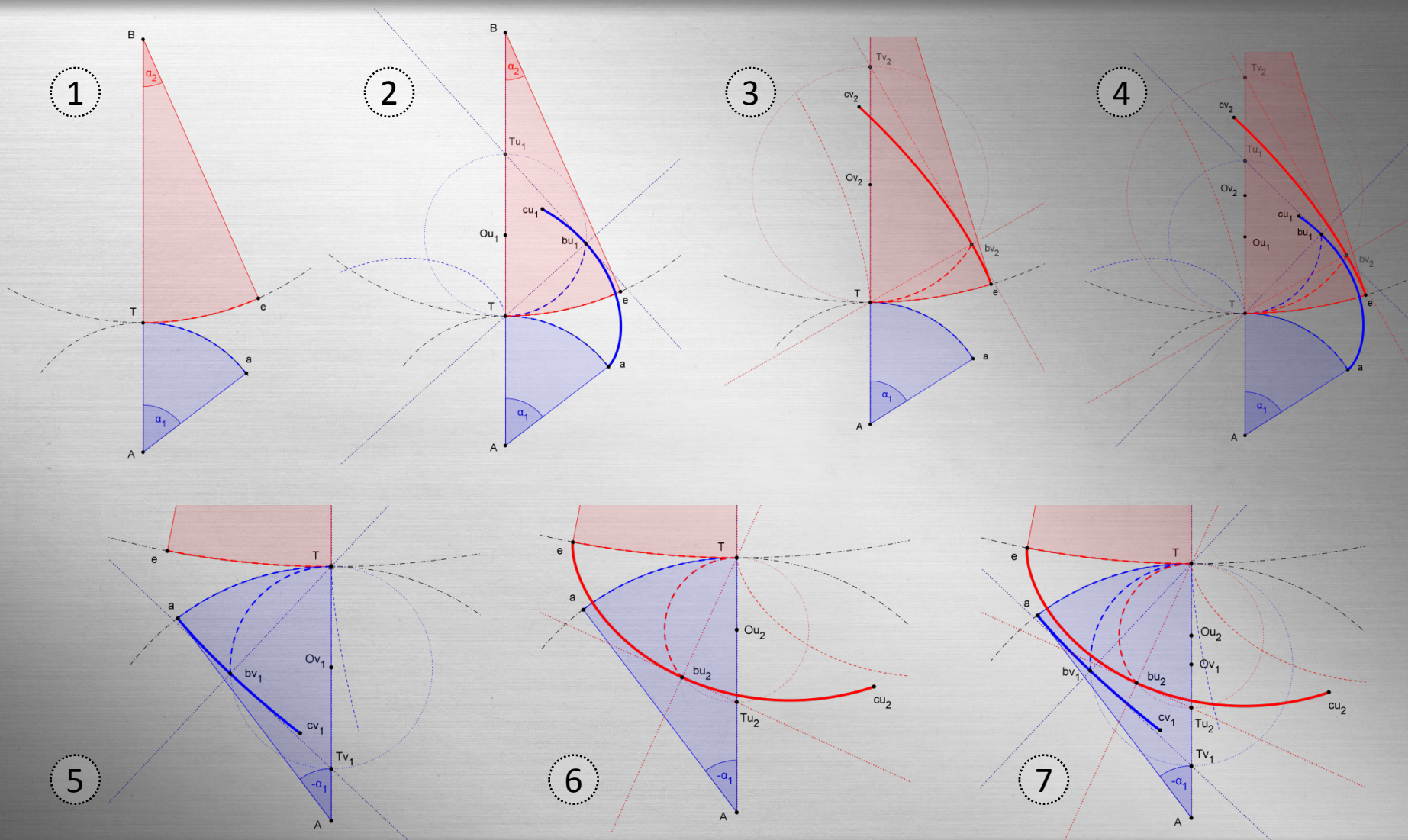
# CYCLOID GEARING





# Cycloid Gearing

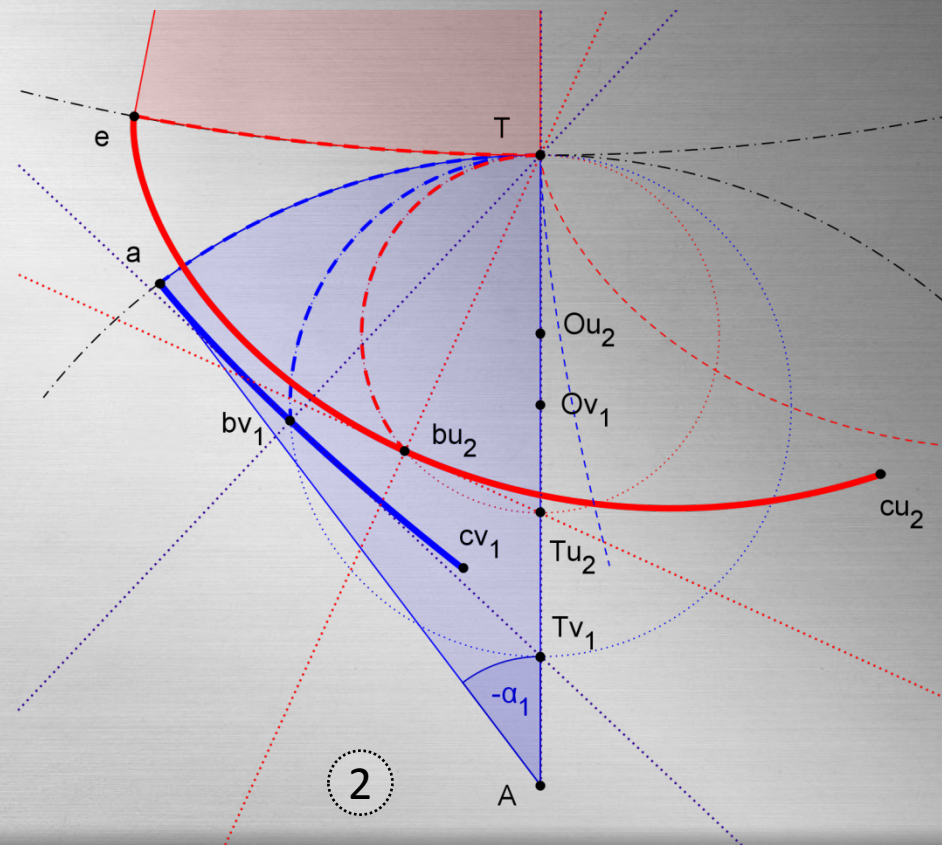
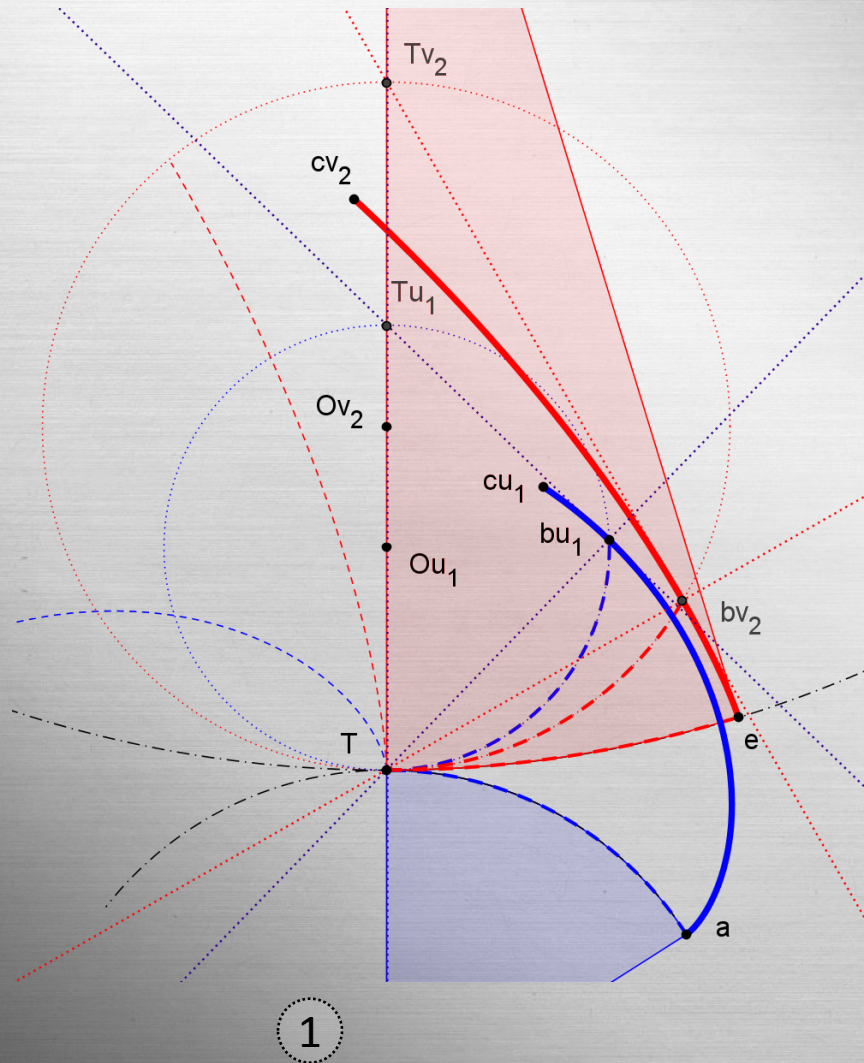
## GeoGebra Laboratory





# Cycloid Gearing

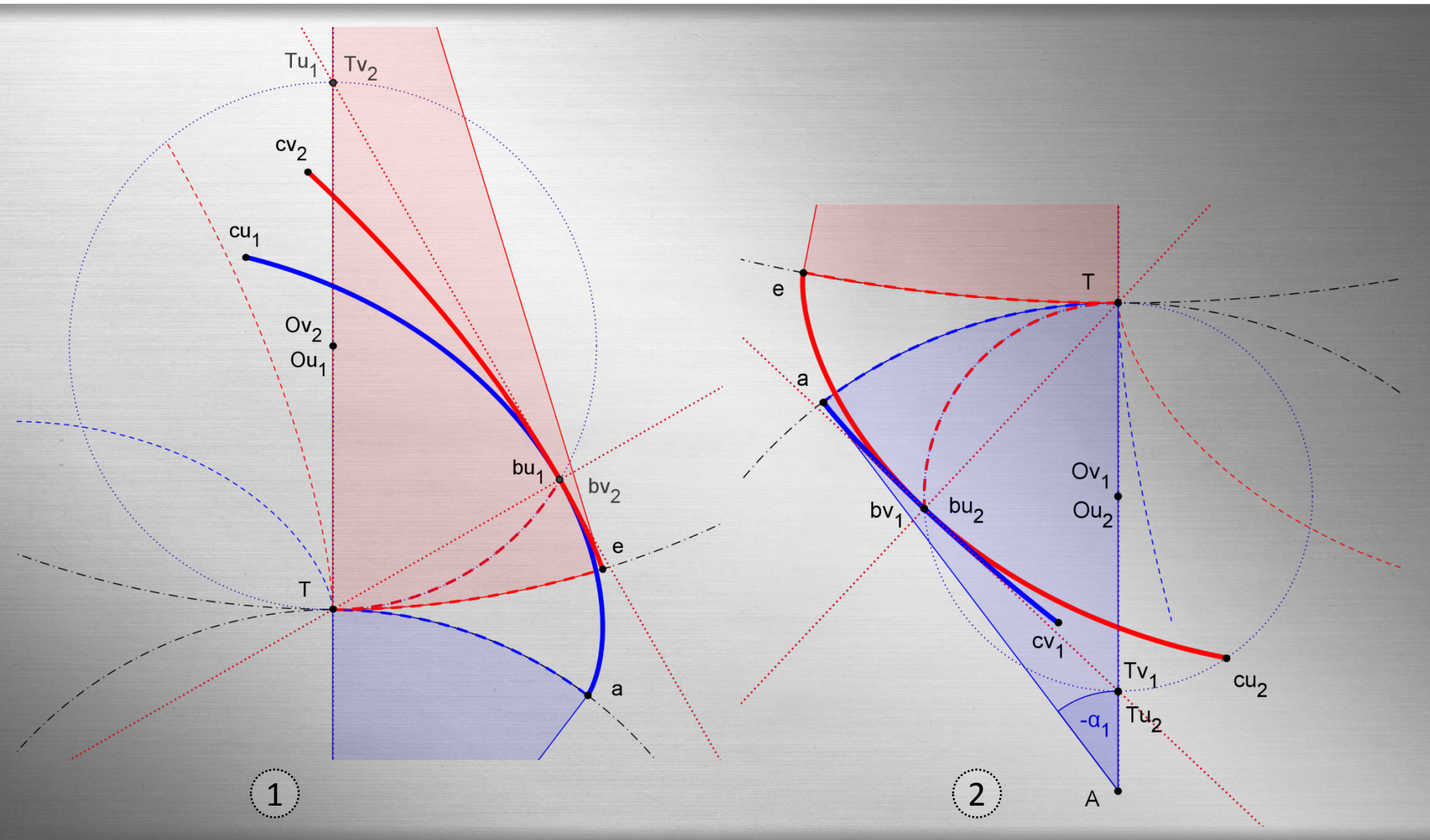
## GeoGebra Laboratory



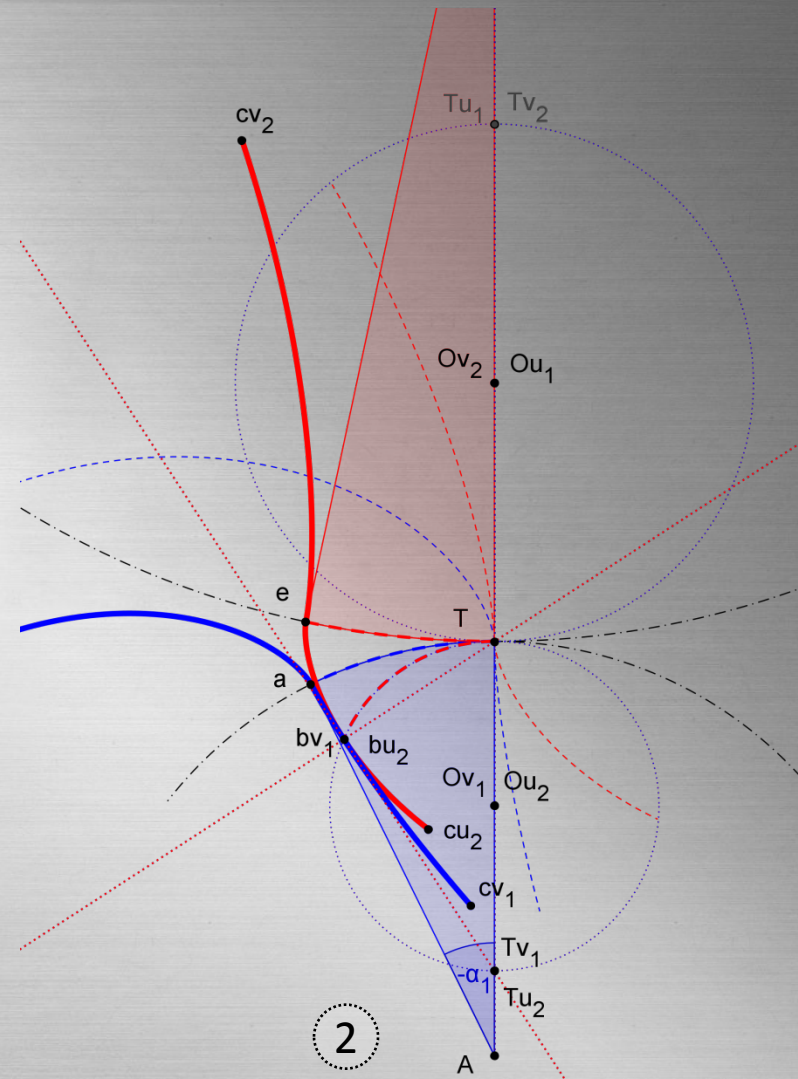


# Cycloid Gearing

## GeoGebra Laboratory

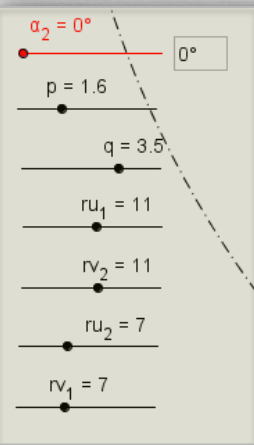






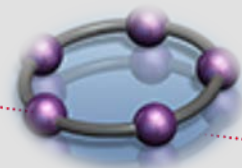
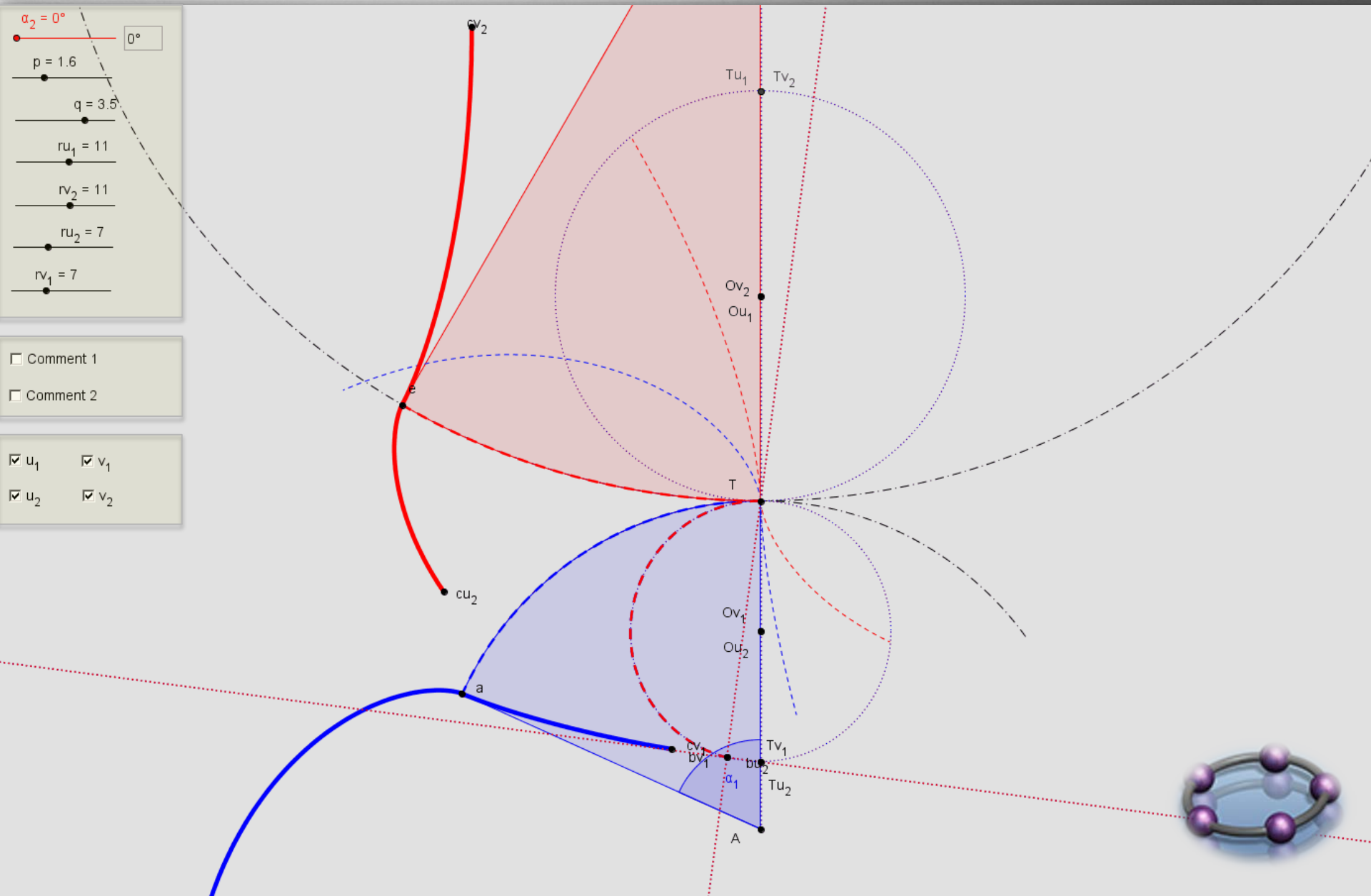


# Cycloid Gearing



☐ Comment 1  
☐ Comment 2

☒  $u_1$     ☒  $v_1$   
☒  $u_2$     ☒  $v_2$





# Cycliod Gearing

## PTC Creo CAD Assembly



Correlation of GeoGebra and CAD software  
in the analysis of cycloid meshing

CADGME 2012

Kaja Maricic, [kjmaricic@gmail.com](mailto:kjmaricic@gmail.com)  
Sima Pastor, [promachinex@gmail.com](mailto:promachinex@gmail.com)

### INPUT PARAMETERS

\*\*\*\*\*

ratio of epicycloid

$p = 1.6$

ratio of hypocycloid

$q = 3.5$

radius of line action

$ru.1 = 11$

$rv.2 = 11$

$ru.2 = 7$

$rv.1 = 7$

duration of curve

$u.1 = 1.1$

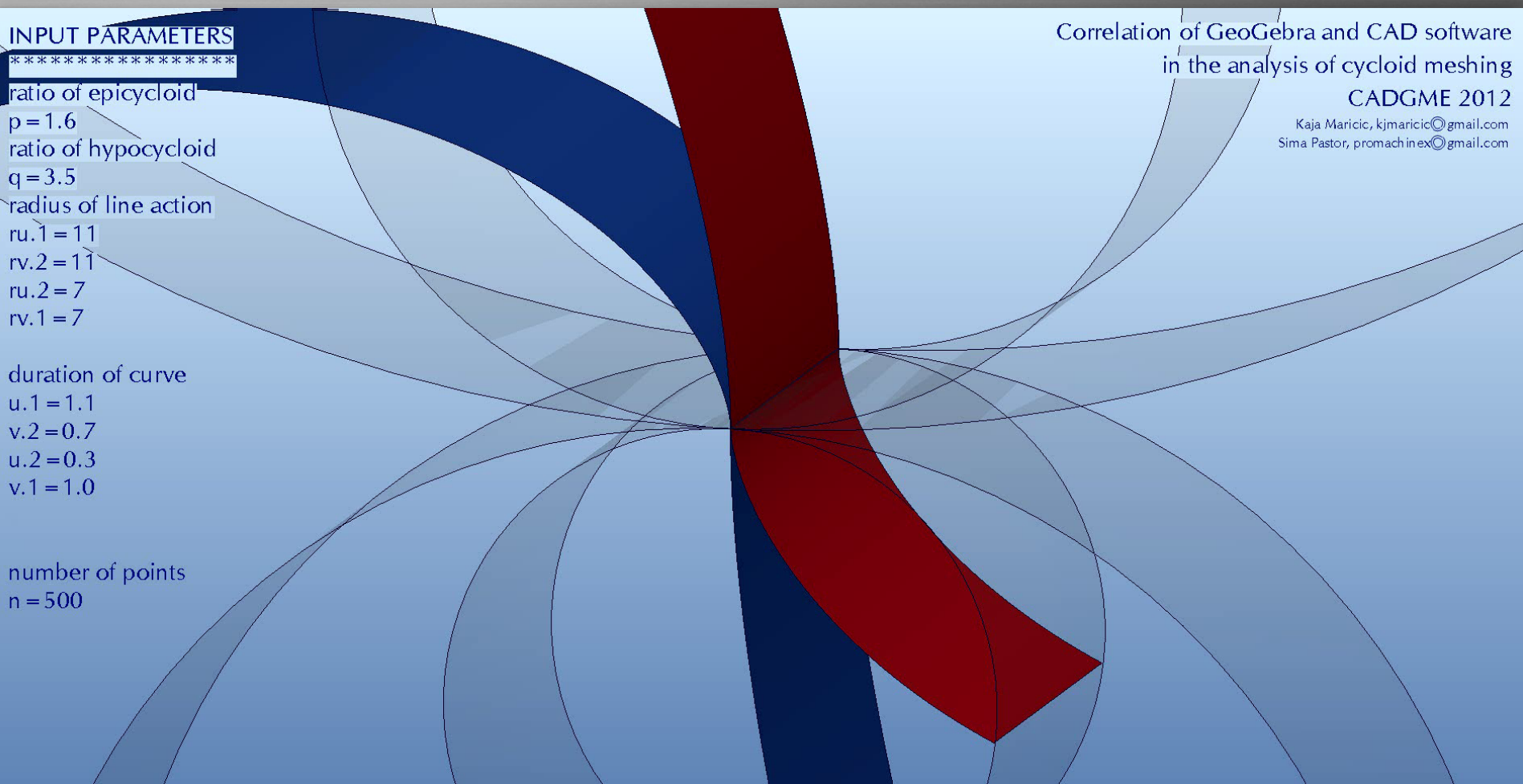
$v.2 = 0.7$

$u.2 = 0.3$

$v.1 = 1.0$

number of points

$n = 500$





# Cycliod Gearing

## PTC Creo CAD Assembly



### INPUT PARAMETERS

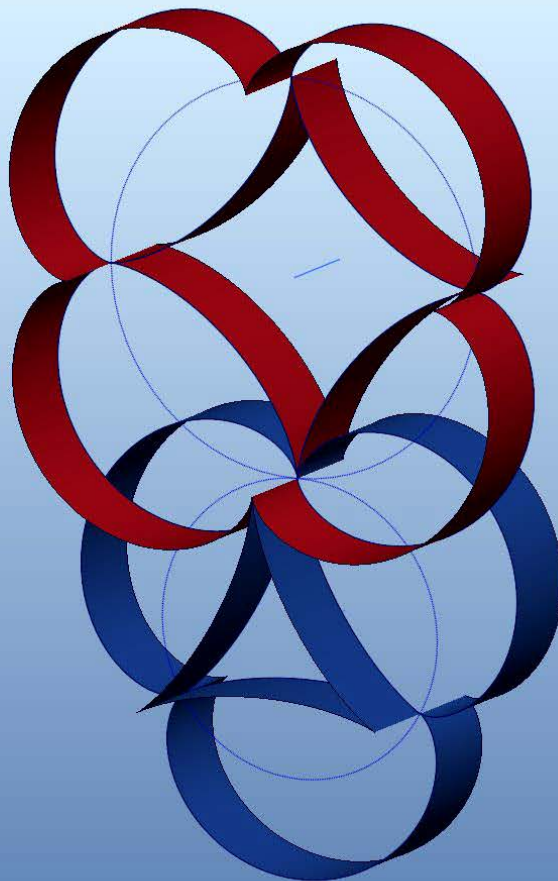
\*\*\*\*\*

ratio of epicycloid  
 $up = 3.0$   
ratio of hypocycloid  
 $vq = 4.0$   
radius of line action  
 $ru.1 = 10$   
 $rv.2 = 10$   
 $ru.2 = 10$   
 $rv.1 = 10$   
duration of curve  
 $u.1 = 6.3$   
 $v.2 = 6.3$   
 $u.2 = 6.3$   
 $v.1 = 6.3$   
number of points  
 $n = 500$

### OUTPUT PARAMETERS

\*\*\*\*\*

diameter of circle  
 $d.1 = 60.0$   
 $d.2 = 80.0$   
ratio of epicycloid  
 $vp = 3.0$   
ratio of hypocycloid  
 $uq = 4.0$



Correlation of GeoGebra and CAD software  
in the analysis of cycloid meshing

CADGME 2012

Kaja Maricic, [kjmaricic@gmail.com](mailto:kjmaricic@gmail.com)  
Sima Pastor, [promachinex@gmail.com](mailto:promachinex@gmail.com)





**Thank you!**  
**Questions...**





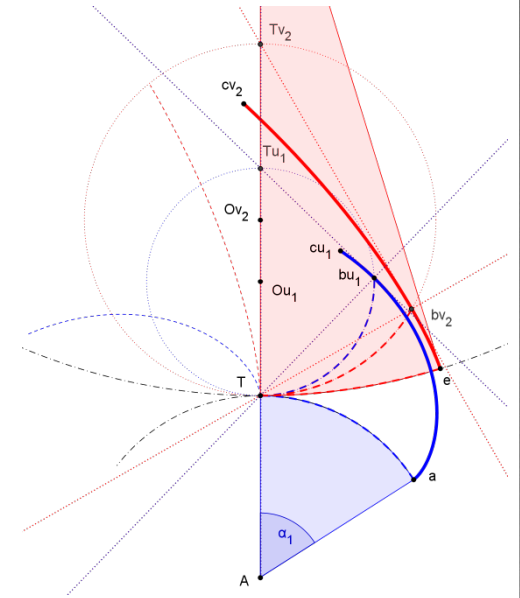
## Contact

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**CADGME 2012**

Novi Sad, Serbia  
22-24 June 2012

**Correlation of GeoGebra and CAD software  
in the analysis of cycloid meshing, Maricic, Pastor**