Tense operators on orthocomplemented posets

Jan Paseka
Department of Mathematics and Statistics
Masaryk University
Brno, Czech Republic
paseka@math.muni.cz

Ivan Chajda
Department of Algebra and Geometry
Palacký University Olomouc
Olomouc, Czech Republic
ivan.chajda@upol.cz

Supported by by a bilateral project I 1923-N25 New Perspectives on Residuated Posets financed by Austrian Science Fund (FWF) and the Czech Science Foundation (GACR)

AAA 2015 June 5 - June 7
Novi Sad, Serbia

Outline

Outline

Introduction - tense operators on De Morgan posets

For Boolean algebras, the so-called tense operators were already introduced by Burges. Tense operators express the quantifiers “it is always going to be the case that” and “it has always been the case that” and hence enable us to express the dimension of time in the logic. In this lecture we introduce tense operators on De Morgan posets.

A crucial problem concerning tense operators is their representation. Having a Boolean algebra with tense operators, it is well known that there exists a time frame such that each of these operators can be obtained by their construction for two-element Boolean algebra $\{0,1\}$. We solved this problem with I. Chajda for tense operators on orthocomplemented posets.

Introduction - tense operators on De Morgan posets

For Boolean algebras, the so-called tense operators were already introduced by Burges. Tense operators express the quantifiers “it is always going to be the case that” and “it has always been the case that” and hence enable us to express the dimension of time in the logic. In this lecture we introduce tense operators on De Morgan posets.

A crucial problem concerning tense operators is their representation. Having a Boolean algebra with tense operators, it is well known that there exists a time frame such that each of these operators can be obtained by their construction for two-element Boolean algebra $\{0, 1\}$. We solved this problem with I. Chajda for tense operators on orthocomplemented posets.

Tense operators on $[0,1]$

Tense operators were used to express the dimension of time in logics.

- Let T be a time scale,
- then elements $f(t)$ from $[0,1]^T$ correspond to the evaluation of the validity of the formula f in time.

For a moment, let T be the interval $[0,2.5]$.

Tense operators on $[0,1]$

Tense operators were used to express the dimension of time in logics.

- Let T be a time scale,
- then elements $f(t)$ from $[0,1]^T$ correspond to the evaluation of the validity of the formula f in time.

For a moment, let T be the interval $[0,2.5]$.

Tense operators on $[0,1]$

Tense operators were used to express the dimension of time in logics.

- Let T be a time scale,
- then elements $f(t)$ from $[0,1]^T$ correspond to the evaluation of the validity of the formula f in time.

For a moment, let T be the interval $[0,2.5]$.

Tense operators on $[0,1]$

Tense operators were used to express the dimension of time in logics.

- Let T be a time scale,
- then elements $f(t)$ from $[0,1]^T$ correspond to the evaluation of the validity of the formula f in time.

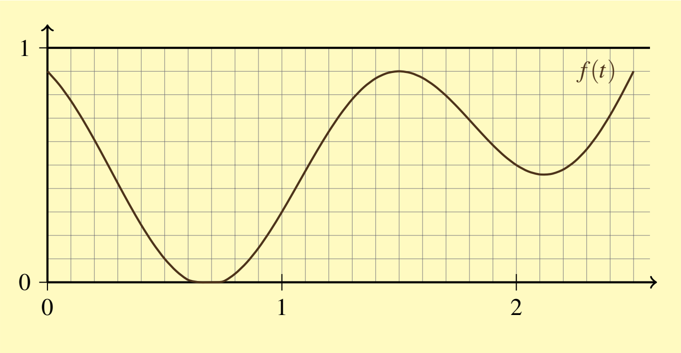
For a moment, let T be the interval $[0,2.5]$.

Tense operators on $[0,1]$

Tense operators were used to express the dimension of time in logics.

- Let T be a time scale,
- then elements $f(t)$ from $[0,1]^T$ correspond to the evaluation of the validity of the formula f in time.

For a moment, let T be the interval $[0,2.5]$.



Tense operators on $[0,1]$

On the time scale T we will introduce a relation $R \subseteq T^2$.

- xRy means that the moment x is before the moment y.

Moreover, we introduce operators G a H on $[0,1]^T$ as follows:

- Gf means that f will be true in future with at least the same degree as f is now.
- Hf means that f was true in past with at least the same degree as f is in present.

Tense operators on $[0,1]$

On the time scale T we will introduce a relation $R \subseteq T^2$.

- xRy means that the moment x is before the moment y.

Moreover, we introduce operators G a H on $[0,1]^T$ as follows:

- Gf means that f will be true in future with at least the same degree as f is now.
- Hf means that f was true in past with at least the same degree as f is in present.

Tense operators on $[0,1]$

On the time scale T we will introduce a relation $R \subseteq T^2$.

- xRy means that the moment x is before the moment y.

Moreover, we introduce operators G a H on $[0,1]^T$ as follows:

- Gf means that f will be true in future with at least the same degree as f is now.
- Hf means that f was true in past with at least the same degree as f is in present.

Tense operators on $[0, 1]$

On the time scale T we will introduce a relation $R \subseteq T^2$.

- xRy means that the moment x is before the moment y.

Moreover, we introduce operators G a H on $[0, 1]^T$ as follows:

- Gf means that f will be true in future with at least the same degree as f is now.
- Hf means that f was true in past with at least the same degree as f is in present.

Tense operators on $[0, 1]$

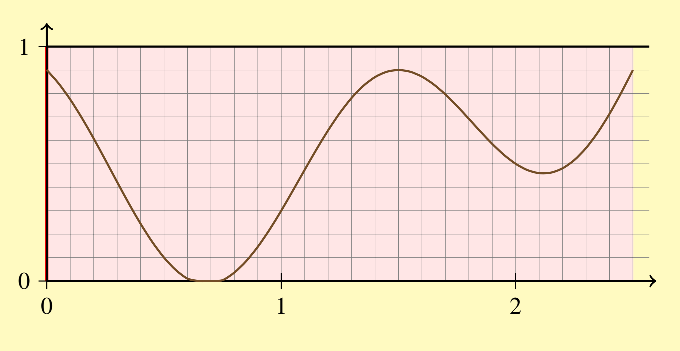
Example: Let R be the relation $\leq$ on the interval $[0, 2.5]$.

Then the operators G and H are introduced as follows.

Tense operators on $[0,1]$

Example: Let R be the relation $\leq$ on the interval $[0,2.5]$.

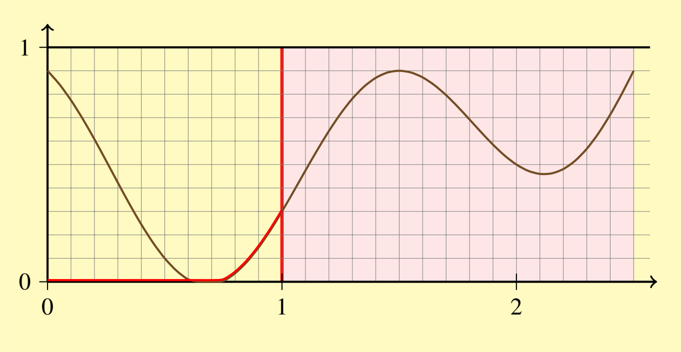
Then the operators G and H are introduced as follows.



Tense operators on $[0, 1]$

Example: Let R be the relation $\leq$ on the interval $[0, 2.5]$.

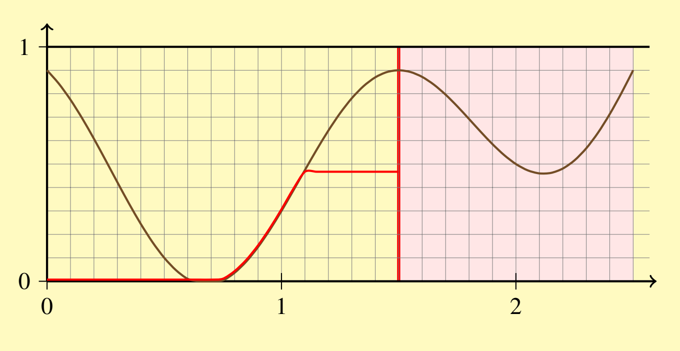
Then the operators G and H are introduced as follows.



Tense operators on $[0,1]$

Example: Let R be the relation $\leq$ on the interval $[0, 2.5]$.

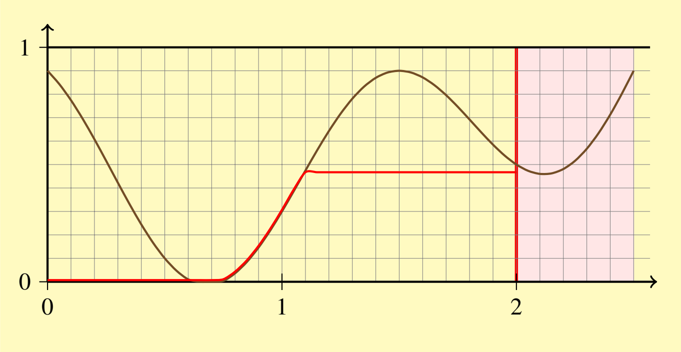
Then the operators G and H are introduced as follows.



Tense operators on $[0,1]$

Example: Let R be the relation $\leq$ on the interval $[0,2.5]$.

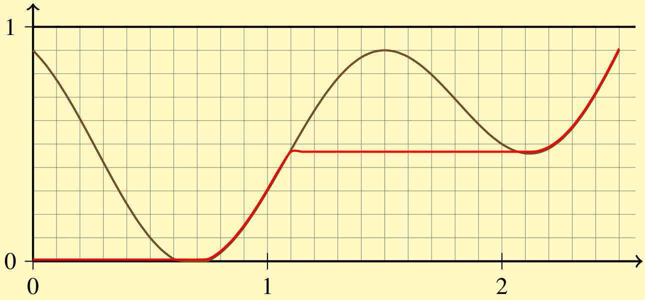
Then the operators G and H are introduced as follows.



Tense operators on $[0,1]$

Example: Let R be the relation $\leq$ on the interval $[0,2.5]$.

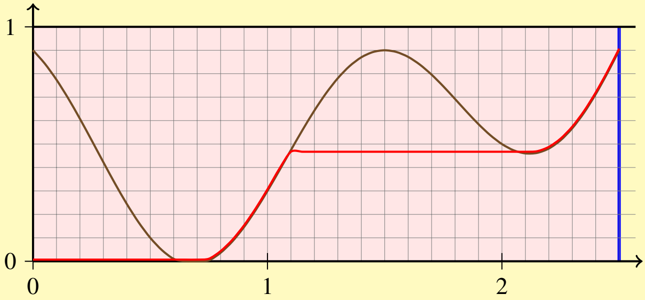
Then the operators G and H are introduced as follows.



Tense operators on $[0, 1]$

Example: Let R be the relation $\leq$ on the interval $[0, 2.5]$.

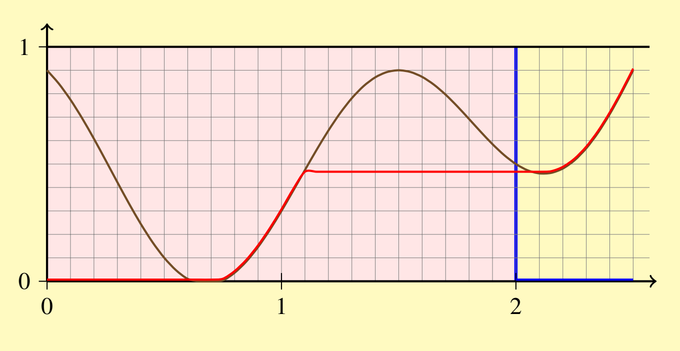
Then the operators G and H are introduced as follows.



Tense operators on $[0,1]$

Example: Let R be the relation $\leq$ on the interval $[0,2.5]$.

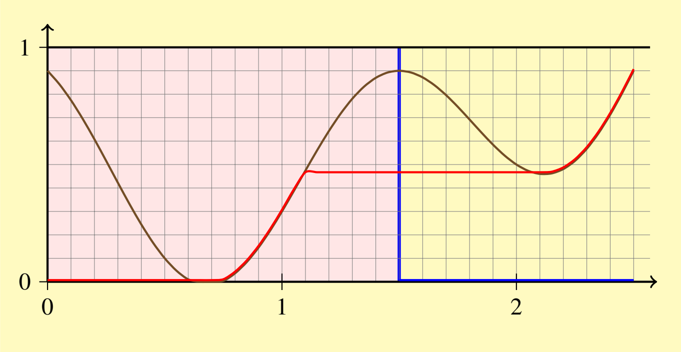
Then the operators G and H are introduced as follows.



Tense operators on $[0, 1]$

Example: Let R be the relation $\leq$ on the interval $[0, 2.5]$.

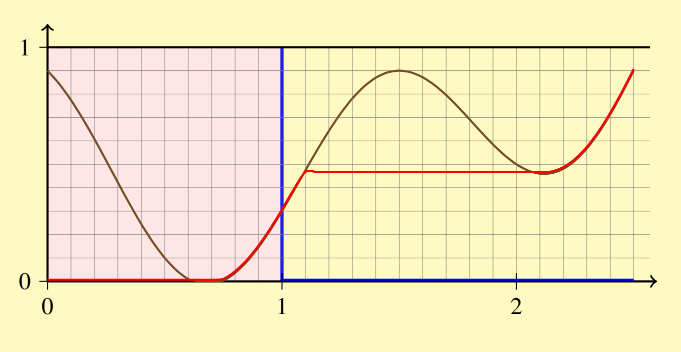
Then the operators G and H are introduced as follows.



Tense operators on $[0,1]$

Example: Let R be the relation $\leq$ on the interval $[0,2.5]$.

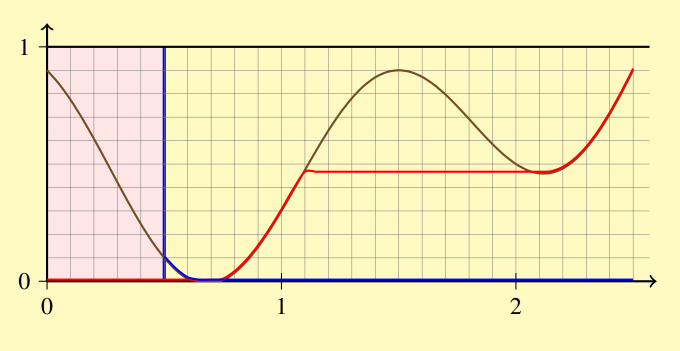
Then the operators G and H are introduced as follows.



Tense operators on $[0,1]$

Example: Let R be the relation $\leq$ on the interval $[0,2.5]$.

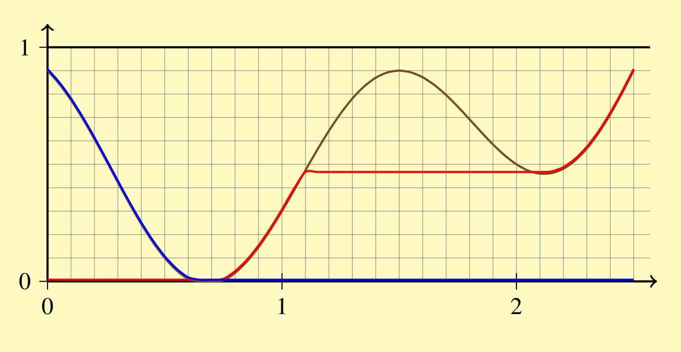
Then the operators G and H are introduced as follows.



Tense operators on $[0,1]$

Example: Let R be the relation $\leq$ on the interval $[0,2.5]$.

Then the operators G and H are introduced as follows.



Outline

Basic definitions - tense operators on De Morgan posets

Definition

By the *tense De Morgan poset* is meant an algebra $(A;\leq,',0,1,G,P,H,F)$ such that $\mathbf{A}=(A;\leq,',0,1)$ is a bounded poset with an antitone involution $'$ (a *De Morgan poset*), (P,G) and (F,G) are Galois connections on A such that, for all $p,q\in A$,

$$G(p)\leq F(p)=G(p')' \quad \text{and} \quad H(q)\leq P(q)=H(q')',$$

G,P,H and F are called *tense operators* on the tense De Morgan poset.

Let $(\mathbf{A}_1;G_1,P_1,H_1,F_1)$ and $(\mathbf{A}_2;G_2,P_2,H_2,F_2)$ be tense De Morgan posets. A *morphism of tense De Morgan posets* is a morphism of De Morgan posets $f:A_1\to A_2$ which simultaneously commutes with the respective tense operators.

A *time frame* is a pair (T,R) where T is a non-empty set and $R\subseteq T\times T$ such that for all $t\in T$ there are $s,u\in T$ with $(s,t),(t,u)\in R$.

Basic definitions - tense operators on De Morgan posets

Definition

By the *tense De Morgan poset* is meant an algebra $(A;\leq,',0,1,G,P,H,F)$ such that $\mathbf{A}=(A;\leq,',0,1)$ is a bounded poset with an antitone involution $'$ (a *De Morgan poset*), (P,G) and (F,G) are Galois connections on A such that, for all $p,q\in A$,

$$G(p)\leq F(p)=G(p')' \quad \text{and} \quad H(q)\leq P(q)=H(q')',$$

G,P,H and F are called *tense operators* on the tense De Morgan poset.

Let $(\mathbf{A}_1;G_1,P_1,H_1,F_1)$ and $(\mathbf{A}_2;G_2,P_2,H_2,F_2)$ be tense De Morgan posets. A *morphism of tense De Morgan posets* is a morphism of De Morgan posets $f:A_1\to A_2$ which simultaneously commutes with the respective tense operators.

A *time frame* is a pair (T,R) where T is a non-empty set and $R\subseteq T\times T$ such that for all $t\in T$ there are $s,u\in T$ with $(s,t),(t,u)\in R$.

Basic results – frames on complete De Morgan lattices

Theorem

Let $\mathbf{M}$ *be a complete De Morgan lattice,* (T,R) *be a time frame,* $\widehat{G},\widehat{P},\widehat{H}$ *and* $\widehat{F}$ *be maps from* M^T *into* M^T *defined by*

$$\begin{aligned}
\widehat{G}(p)(s) &= \bigwedge\{p(t) \mid t\in T, sRt\},\\
\widehat{F}(p)(s) &= \bigvee\{p(t) \mid t\in T, sRt\},\\
\widehat{H}(p)(s) &= \bigwedge\{p(t) \mid t\in T, tRs\},\\
\widehat{P}(p)(s) &= \bigvee\{p(t) \mid t\in T, tRs\}
\end{aligned}$$

for all $p\in M^T$ *and* $s\in T$*. Then* $(\widehat{P},\widehat{G})$ *and* $(\widehat{F},\widehat{H})$ *are Galois connections on* $\mathbf{M}^T$ *such that, for all* $p,q\in M^T$*,*

$$\widehat{G}(p)\leq\widehat{F}(p)=\widehat{G}(p')' \quad \text{and} \quad \widehat{H}(q)\leq\widehat{P}(q)=\widehat{H}(q')'.$$

Hence $\widehat{G},\widehat{P},\widehat{H}$ *and* $\widehat{F}$ *are* tense operators *in our sense.*

Basic results – frames on complete De Morgan lattices

Theorem

Let $\mathbf{M}$ *be a complete De Morgan lattice,* (T,R) *be a time frame,* $\widehat{G},\widehat{P},\widehat{H}$ *and* $\widehat{F}$ *be maps from* M^T *into* M^T *defined by*

$$\begin{array}{rcl} \widehat{G}(p)(s) & = & \bigwedge\{p(t) \mid t \in T, sRt\}, \\ \widehat{F}(p)(s) & = & \bigvee\{p(t) \mid t \in T, sRt\}, \\ \widehat{H}(p)(s) & = & \bigwedge\{p(t) \mid t \in T, tRs\}, \\ \widehat{P}(p)(s) & = & \bigvee\{p(t) \mid t \in T, tRs\} \end{array}$$

for all $p \in M^T$ *and* $s \in T$*. Then* $(\widehat{P},\widehat{G})$ *and* $(\widehat{F},\widehat{H})$ *are Galois connections on* $\mathbf{M}^T$ *such that, for all* $p,q \in M^T$*,*

$$\widehat{G}(p) \leq \widehat{F}(p) = \widehat{G}(p')' \quad \text{and} \quad \widehat{H}(q) \leq \widehat{P}(q) = \widehat{H}(q')'.$$

Hence $\widehat{G},\widehat{P},\widehat{H}$ *and* $\widehat{F}$ *are* tense operators *in our sense.*

Basic results – frames on complete De Morgan lattices

Theorem

Let $\mathbf{M}$ *be a complete De Morgan lattice,* (T,R) *be a time frame,* $\widehat{G},\widehat{P},\widehat{H}$ *and* $\widehat{F}$ *be maps from* M^T *into* M^T *defined by*

$$\begin{array}{rcl} \widehat{G}(p)(s) & = & \bigwedge\{p(t) \mid t \in T, sRt\}, \\ \widehat{F}(p)(s) & = & \bigvee\{p(t) \mid t \in T, sRt\}, \\ \widehat{H}(p)(s) & = & \bigwedge\{p(t) \mid t \in T, tRs\}, \\ \widehat{P}(p)(s) & = & \bigvee\{p(t) \mid t \in T, tRs\} \end{array}$$

for all $p \in M^T$ *and* $s \in T$*. Then* $(\widehat{P},\widehat{G})$ *and* $(\widehat{F},\widehat{H})$ *are Galois connections on* $\mathbf{M}^T$ *such that, for all* $p,q \in M^T$*,*

$$\widehat{G}(p) \leq \widehat{F}(p) = \widehat{G}(p')' \quad \text{and} \quad \widehat{H}(q) \leq \widehat{P}(q) = \widehat{H}(q')'.$$

Hence $\widehat{G},\widehat{P},\widehat{H}$ *and* $\widehat{F}$ *are* tense operators *in our sense.*

Outline

Definition

Let P,Q be bounded posets and let T be a set of order-preserving maps from P to Q. Then T is called *order reflecting* or a *full set* if

$$((\forall s \in T)\, s(a) \le s(b)) \implies a \le b$$

for any elements $a,b \in P$.

Using an approach due to Katrnoška and Marlow we are able to represent tense De Morgan posets $\mathbf{A}$ which are orthocomplemented, i.e.,

$$x \wedge x' = 0$$

for all $x \in A$.

Definition

Katrnoška, Marlow Let $\mathbf{A} = (A; \le, ', 0, 1)$ be an orthocomplemented poset, $B \subseteq A$. The set B is called an *M-base* of $\mathbf{A}$ if B is an upper set of $\mathbf{A}$ such that, for all $a \in A$, $a \in B$ if and only if $a' \notin B$.

Definition

Let P, Q be bounded posets and let T be a set of order-preserving maps from P to Q. Then T is called *order reflecting* or a *full set* if

$$((\forall s \in T)\, s(a) \leq s(b)) \implies a \leq b$$

for any elements $a, b \in P$.

Using an approach due to Katrnoška and Marlow we are able to represent tense De Morgan posets $\mathbf{A}$ which are orthocomplemented, i.e.,

$$x \wedge x' = 0$$

for all $x \in A$.

Definition

Katrnoška, Marlow Let $\mathbf{A} = (A; \leq, ', 0, 1)$ be an orthocomplemented poset, $B \subseteq A$. The set B is called an *M-base* of $\mathbf{A}$ if B is an upper set of $\mathbf{A}$ such that, for all $a \in A$, $a \in B$ if and only if $a' \notin B$.

The following lemma is essentially due to Katrnoška.

Lemma

Let $\mathbf{A} = (A; \leq, ', 0, 1)$ *be an orthocomplemented poset. Then the following holds.*

(i) *If* $a \in A, a \neq 0$ *then there is always such an M-base* B *of* $\mathbf{A}$ *such that* $a \in B$.

(ii) *If* $a \in A, a \neq 0$ *and* B *is an M-base of* $\mathbf{A}$ *such that* $a \notin B$ *then the set* $D = (B \setminus (a']) \cup [a)$ *is an M-base of* $\mathbf{A}$ *such that* $a \in D$.

(iii) *If* $a \in A, a \neq 0$ *and* $V \subseteq A$ *is an upper subset of* $\mathbf{A}$ *such that* $a \notin V$ *and* $b \in V$ *implies* $b' \notin V$ *then there is always such an M-base* B *of* $\mathbf{A}$ *such that* $V \subseteq B$ *and* $a \notin B$.

(iv) *If* B *is an M-base of* $\mathbf{A}$ *and* $\mathbf{2} = (\{0,1\}; \leq, ', 0, 1)$ *is an orthocomplemented poset then the map* $g_B : A \to \{0,1\}$ *defined by the prescription* $g_B(a) = 1$ *iff* $a \in B$ *is a morphism of De Morgan posets. Conversely, any morphism* $g : A \to \{0,1\}$ *from* $\mathbf{A}$ *to* $\mathbf{2}$ *of De Morgan posets defines an M-base* $B_g = \{x \in A \mid g(x) = 1\}$.

The following lemma is essentially due to Katrnoška.

Lemma

Let $\mathbf{A} = (A; \leq, ', 0, 1)$ *be an orthocomplemented poset. Then the following holds.*

(i) *If* $a \in A, a \neq 0$ *then there is always such an M-base* B *of* $\mathbf{A}$ *such that* $a \in B$.

(ii) *If* $a \in A, a \neq 0$ *and* B *is an M-base of* $\mathbf{A}$ *such that* $a \notin B$ *then the set* $D = (B \setminus (a']) \cup [a)$ *is an M-base of* $\mathbf{A}$ *such that* $a \in D$.

(iii) *If* $a \in A, a \neq 0$ *and* $V \subseteq A$ *is an upper subset of* $\mathbf{A}$ *such that* $a \notin V$ *and* $b \in V$ *implies* $b' \notin V$ *then there is always such an M-base* B *of* $\mathbf{A}$ *such that* $V \subseteq B$ *and* $a \notin B$.

(iv) *If* B *is an M-base of* $\mathbf{A}$ *and* $\mathbf{2} = (\{0,1\}; \leq, ', 0, 1)$ *is an orthocomplemented poset then the map* $g_B : A \to \{0,1\}$ *defined by the prescription* $g_B(a) = 1$ *iff* $a \in B$ *is a morphism of De Morgan posets. Conversely, any morphism* $g : A \to \{0,1\}$ *from* $\mathbf{A}$ *to* $\mathbf{2}$ *of De Morgan posets defines an M-base* $B_g = \{x \in A \mid g(x) = 1\}$.

The following lemma is essentially due to Katrnoška.

Lemma

Let $\mathbf{A} = (A; \leq, ', 0, 1)$ be an orthocomplemented poset. Then the following holds.

(i) *If $a \in A, a \neq 0$ then there is always such an M-base B of $\mathbf{A}$ such that $a \in B$.*

(ii) *If $a \in A, a \neq 0$ and B is an M-base of $\mathbf{A}$ such that $a \notin B$ then the set $D = (B \setminus (a']) \cup [a)$ is an M-base of $\mathbf{A}$ such that $a \in D$.*

(iii) *If $a \in A, a \neq 0$ and $V \subseteq A$ is an upper subset of $\mathbf{A}$ such that $a \notin V$ and $b \in V$ implies $b' \notin V$ then there is always such an M-base B of $\mathbf{A}$ such that $V \subseteq B$ and $a \notin B$.*

(iv) *If B is an M-base of $\mathbf{A}$ and $\mathbf{2} = (\{0,1\}; \leq, ', 0, 1)$ is an orthocomplemented poset then the map $g_B : A \to \{0,1\}$ defined by the prescription $g_B(a) = 1$ iff $a \in B$ is a morphism of De Morgan posets. Conversely, any morphism $g : A \to \{0,1\}$ from $\mathbf{A}$ to $\mathbf{2}$ of De Morgan posets defines an M-base $B_g = \{x \in A \mid g(x) = 1\}$.*

The following lemma is essentially due to Katrnoška.

Lemma

Let $\mathbf{A} = (A; \leq, ', 0, 1)$ *be an orthocomplemented poset. Then the following holds.*

(i) *If* $a \in A, a \neq 0$ *then there is always such an M-base* B *of* $\mathbf{A}$ *such that* $a \in B$*.*

(ii) *If* $a \in A, a \neq 0$ *and* B *is an M-base of* $\mathbf{A}$ *such that* $a \notin B$ *then the set* $D = (B \setminus (a']) \cup [a)$ *is an M-base of* $\mathbf{A}$ *such that* $a \in D$*.*

(iii) *If* $a \in A, a \neq 0$ *and* $V \subseteq A$ *is an upper subset of* $\mathbf{A}$ *such that* $a \notin V$ *and* $b \in V$ *implies* $b' \notin V$ *then there is always such an M-base* B *of* $\mathbf{A}$ *such that* $V \subseteq B$ *and* $a \notin B$*.*

(iv) *If* B *is an M-base of* $\mathbf{A}$ *and* $\mathbf{2} = (\{0,1\}; \leq, ', 0, 1)$ *is an orthocomplemented poset then the map* $g_B : A \to \{0,1\}$ *defined by the prescription* $g_B(a) = 1$ *iff* $a \in B$ *is a morphism of De Morgan posets. Conversely, any morphism* $g : A \to \{0,1\}$ *from* $\mathbf{A}$ *to* $\mathbf{2}$ *of De Morgan posets defines an M-base* $B_g = \{x \in A \mid g(x) = 1\}$*.*

The following lemma is essentially due to Katrnoška.

Lemma

Let $\mathbf{A} = (A; \leq, ', 0, 1)$ be an orthocomplemented poset. Then the following holds.

(i) *If $a \in A, a \neq 0$ then there is always such an M-base B of $\mathbf{A}$ such that $a \in B$.*

(ii) *If $a \in A, a \neq 0$ and B is an M-base of $\mathbf{A}$ such that $a \notin B$ then the set $D = (B \setminus (a']) \cup [a)$ is an M-base of $\mathbf{A}$ such that $a \in D$.*

(iii) *If $a \in A, a \neq 0$ and $V \subseteq A$ is an upper subset of $\mathbf{A}$ such that $a \notin V$ and $b \in V$ implies $b' \notin V$ then there is always such an M-base B of $\mathbf{A}$ such that $V \subseteq B$ and $a \notin B$.*

(iv) *If B is an M-base of $\mathbf{A}$ and $\mathbf{2} = (\{0, 1\}; \leq, ', 0, 1)$ is an orthocomplemented poset then the map $g_B : A \to \{0, 1\}$ defined by the prescription $g_B(a) = 1$ iff $a \in B$ is a morphism of De Morgan posets. Conversely, any morphism $g : A \to \{0, 1\}$ from $\mathbf{A}$ to $\mathbf{2}$ of De Morgan posets defines an M-base $B_g = \{x \in A \mid g(x) = 1\}$.*

Let us denote by $T_{\mathbf{A}}^{\text{orth}}$ the set of all morphisms of De Morgan posets into the two-element orthocomplemented poset $\mathbf{2} = (\{0,1\}; \leq, ', 0, 1)$. Note that any element s of $T_{\mathbf{A}}^{\text{orth}}$ is of the form g_B for a suitable M-base B.
The following result by Katrnoška is well known.

Proposition

Let $\mathbf{A} = (A; \leq, ', 0, 1)$ be an orthocomplemented poset. Then the map $i_{\mathbf{A}} : A \to 2^{T_{\mathbf{A}}^{orth}}$ constructed by $i_{\mathbf{A}}(a)(s) = s(a)$ for all $a \in A$ and all $s \in T_{\mathbf{A}}^{orth}$ is an order reflecting morphism of De Morgan posets such that $i_{\mathbf{A}}(A)$ is a sub-De Morgan poset of $\mathbf{2}^{T_{\mathbf{A}}^{orth}}$.

The representation theorem for order-preserving morphisms

Theorem

Let $\mathbf{A} = (A; \leq, ', 0, 1)$ *and* $\mathbf{B} = (B; \leq, ', 0, 1)$ *be orthocomplemented posets. Let* $P : A \to B$ *and* $G : B \to A$ *be morphisms of posets such that* $P(x)' \leq P(x')$, $G(x') \leq G(x)'$, $P(0) = 0$, $G(1) = 1$. *Let* $R_G \subseteq T_{\mathbf{A}}^{orth} \times T_{\mathbf{B}}^{orth}$ *be the* G*-induced relation by* $\mathbf{2}$ *and* $R^P \subseteq T_{\mathbf{A}}^{orth} \times T_{\mathbf{B}}^{orth}$ *be the* P*-induced relation by* $\mathbf{2}$ *defined as follows:*

$$R_G = \{(s,t) \in S \times T \mid (\forall b \in B)(s(G(b)) \leq t(b))\} \qquad (\dagger)$$

and

$$R^P = \{(s,t) \in S \times T \mid (\forall a \in A)(s(a) \leq t(P(a)))\}. \qquad (\dagger\dagger)$$

Then the maps $i_{\mathbf{A}}^{T_{\mathbf{A}}^{orth}}$ *and* $i_{\mathbf{B}}^{T_{\mathbf{B}}^{orth}}$ *are order reflecting morphisms of De Morgan posets into the complete orthocomplemented lattices* $\mathbf{2}^{T_{\mathbf{A}}^{orth}}$ *and* $\mathbf{2}^{T_{\mathbf{B}}^{orth}}$ *such that* $\widetilde{P} \circ i_{\mathbf{A}}^{T_{\mathbf{A}}^{orth}} = i_{\mathbf{B}}^{T_{\mathbf{B}}^{orth}} \circ P$ *and* $\widehat{G} \circ i_{\mathbf{B}}^{T_{\mathbf{B}}^{orth}} = i_{\mathbf{A}}^{T_{\mathbf{A}}^{orth}} \circ G$ *where* $\widehat{G}$ *or* $\widetilde{P}$ *are constructed by means of* R_G *or* R^P*, respectively.*
In particular, if (P, G) *is a Galois connection then* $R_G = R^P$, $\widetilde{P} = \widehat{P}$ *and* $\widetilde{G} = \widehat{G}$, *where* $\widehat{P}$ *or* $\widetilde{G}$ *are constructed by means of* R_G *or* R^P*, respectively.*

The representation theorem for order-preserving morphisms

Theorem

Equivalently, the following diagrams commute:

$$
\begin{array}{ccc}
A & \xrightarrow{\quad P \quad} & B \\
\Big\downarrow {\scriptstyle i_{\mathbf{A}}^{T_{\mathbf{A}}^{orth}}} & & \Big\downarrow {\scriptstyle i_{\mathbf{B}}^{T_{\mathbf{B}}^{orth}}} \\
2^{T_{\mathbf{A}}^{orth}} & \xrightarrow[\quad \widetilde{P} \quad]{} & 2^{T_{\mathbf{B}}^{orth}}
\end{array}
\qquad
\begin{array}{ccc}
B & \xrightarrow{\quad G \quad} & A \\
\Big\downarrow {\scriptstyle i_{\mathbf{B}}^{T_{\mathbf{B}}^{orth}}} & & \Big\downarrow {\scriptstyle i_{\mathbf{A}}^{T_{\mathbf{A}}^{orth}}} \\
2^{T_{\mathbf{B}}^{orth}} & \xrightarrow[\quad \widehat{G} \quad]{} & 2^{T_{\mathbf{A}}^{orth}}
\end{array}
.
$$

The representation theorem for Galois connections

As promised above we will establish a set representation of tense orthocomplemented posets.

Theorem

(Set representation theorem for tense orthocomplemented posets) *Let $(\mathbf{A}; G, P, H, F)$ be a tense orthocomplemented poset and R_G the G-induced relation on $T_{\mathbf{A}}^{orth}$ by $\mathbf{2}$. Then the map $i_{\mathbf{A}}^{T_{\mathbf{A}}^{orth}}$ is an order reflecting morphism of tense De Morgan posets into the complete tense orthocomplemented poset $(\mathbf{2}^{T_{\mathbf{A}}^{orth}}; \widehat{G}, \widehat{P}, \widehat{H}, \widehat{F})$ constructed by the time frame $(T_{\mathbf{A}}^{orth}, R_G)$.*

References

- J. Burges, Basic tense logic, in: Handbook of Philosophical Logic, vol. II (D. M. Gabbay, F. Günther, eds.), D. Reidel Publ. Comp., 1984, pp. 89–139.
- F. Katrnoška, On the representation of orthocomplemented posets, Commentationes Mathematicae Universitatis Carolinae **23** (1982), 489–498.
- A.R.Marlow, Quantum theory and Hilbert space, Journal of Math. Phys. **19** (1978), 1841–1846.

References

- J. Burges, Basic tense logic, in: Handbook of Philosophical Logic, vol. II (D. M. Gabbay, F. Günther, eds.), D. Reidel Publ. Comp., 1984, pp. 89–139.
- F. Katrnoška, On the representation of orthocomplemented posets, Commentationes Mathematicae Universitatis Carolinae **23** (1982), 489–498.
- A.R.Marlow, Quantum theory and Hilbert space, Journal of Math. Phys. **19** (1978), 1841–1846.

References

- J. Burges, Basic tense logic, in: Handbook of Philosophical Logic, vol. II (D. M. Gabbay, F. Günther, eds.), D. Reidel Publ. Comp., 1984, pp. 89–139.
- F. Katrnoška, On the representation of orthocomplemented posets, Commentationes Mathematicae Universitatis Carolinae **23** (1982), 489–498.
- A.R.Marlow, Quantum theory and Hilbert space, Journal of Math. Phys. **19** (1978), 1841–1846.

Thank you for your attention.